1 Natural Induction on Inequality

Note 3

Prove that if $n \in \mathbb{N}$ and $x > 0$, then $(1+x)^n \geq 1+nx$.

2 Make It Stronger

Note 3

Suppose that the sequence $a_1, a_2, \ldots$ is defined by $a_1 = 1$ and $a_{n+1} = 3a_n^2$ for $n \geq 1$. We want to prove that

$$a_n \leq 3^{(2^n)}$$

for every positive integer n.

(a) Suppose that we want to prove this statement using induction. Can we let our inductive hypothesis be simply $a_n \leq 3^{(2^n)}$? Attempt an induction proof with this hypothesis to show why this does not work.

(b) Try to instead prove the statement $a_n \leq 3^{(2^n-1)}$ using induction.

(c) Why does the hypothesis in part (b) imply the overall claim?

3 Binary Numbers

Note 3

Prove that every positive integer n can be written in binary. In other words, prove that for any positive integer n, we can write

$$n = c_k \cdot 2^k + c_{k-1} \cdot 2^{k-1} + \cdots + c_1 \cdot 2^1 + c_0 \cdot 2^0,$$

for some $k \in \mathbb{N}$ and $c_i \in \{0,1\}$ for all $i \leq k$.

4 Fibonacci for Home

Note 3

Recall, the Fibonacci numbers, defined recursively as

$F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-2} + F_{n-1}$.

Prove that every third Fibonacci number is even. For example, $F_3 = 2$ is even and $F_6 = 8$ is even.