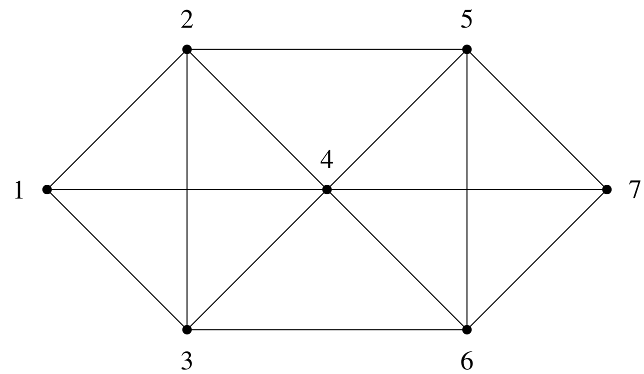
CS 70	Discrete Mathematics and Probability Theory	
Fall 2023	Rao, Tal	DIS 2B

1 Eulerian Tour and Eulerian Walk

Note 5



(a) Is there an Eulerian tour in the graph above? If no, give justification. If yes, provide an example.

(b) Is there an Eulerian walk in the graph above? An Eulerian walk is a walk that uses each edge exactly once. If no, give justification. If yes, provide an example.

(c) What is the condition that there is an Eulerian walk in an undirected graph? Briefly justify your answer.

2 Coloring Trees

Note 5

(a) Prove that all trees with at least 2 vertices have at least two leaves. Recall that a leaf is defined as a node in a tree with degree exactly 1.

(b) Prove that all trees with at least 2 vertices are *bipartite*: the vertices can be partitioned into two groups so that every edge goes between the two groups.

[*Hint:* Use induction on the number of vertices.]

3 Not everything is normal: Odd-Degree Vertices

Note 5

Claim: Let $G = (V, E)$ be an undirected graph. The number of vertices of G that have odd degree is even.

Prove the claim above using:

(i) Direct proof (e.g., counting the number of edges in G). *Hint: in lecture, we proved that* $\sum_{v \in V} \deg v = 2|E|$.

(ii) Induction on $m = |E|$ (number of edges)

(iii) Induction on $n = |V|$ (number of vertices)