

## 1 Venn Diagram

Note 13

Out of 1,000 computer science students, 400 belong to a club (and may work part time), 500 work part time (and may belong to a club), and 50 belong to a club and work part time.

- (a) Suppose we choose a student uniformly at random. Let  $C$  be the event that the student belongs to a club and  $P$  the event that the student works part time. Draw a picture of the sample space  $\Omega$  and the events  $C$  and  $P$ .
- (b) What is the probability that the student belongs to a club?
- (c) What is the probability that the student works part time?
- (d) What is the probability that the student belongs to a club AND works part time?
- (e) What is the probability that the student belongs to a club OR works part time?

## 2 Flippin' Coins

Note 13

Suppose we have an unbiased coin, with outcomes  $H$  and  $T$ , with probability of heads  $\mathbb{P}[H] = 1/2$  and probability of tails also  $\mathbb{P}[T] = 1/2$ . Suppose we perform an experiment in which we toss the coin 3 times. An outcome of this experiment is  $(X_1, X_2, X_3)$ , where  $X_i \in \{H, T\}$ .

- (a) What is the *sample space* for our experiment?
- (b) Which of the following are examples of *events*? Select all that apply.
- $\{(H, H, T), (H, H), (T)\}$
  - $\{(T, H, H), (H, T, H), (H, H, T), (H, H, H)\}$
  - $\{(T, T, T)\}$
  - $\{(T, T, T), (H, H, H)\}$
  - $\{(T, H, T), (H, H, T)\}$
- (c) What is the complement of the event  $\{(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, T, T)\}$ ?
- (d) Let  $A$  be the event that our outcome has 0 heads. Let  $B$  be the event that our outcome has exactly 2 heads. What is  $A \cup B$ ?
- (e) What is the probability of the outcome  $(H, H, T)$ ?
- (f) What is the probability of the event that our outcome has exactly two heads?
- (g) What is the probability of the event that our outcome has at least one head?

### 3 Sampling

Note 13

Suppose you have balls numbered  $1, \dots, n$ , where  $n$  is a positive integer  $\geq 2$ , inside a coffee mug. You pick a ball uniformly at random, look at the number on the ball, replace the ball back into the coffee mug, and pick another ball uniformly at random.

- (a) What is the probability that the first ball is 1 and the second ball is 2?
  
- (b) What is the probability that the second ball's number is strictly less than the first ball's number?
  
- (c) What is the probability that the second ball's number is exactly one greater than the first ball's number?
  
- (d) Now, assume that after you looked at the first ball, you did *not* replace the ball in the coffee mug (instead, you threw the ball away), and then you drew a second ball as before. Now, what are the answers to the previous parts?