## 1 Head Count

Note 15 Consider a coin with $\mathbb{P}[$ Heads $]=2 / 5$. Suppose you flip the coin 20 times, and define $X$ to be the number of heads.
(a) What is $\mathbb{P}[X=k]$, for some $0 \leq k \leq 20$ ?
(b) Name the distribution of $X$ and what its parameters are.
(c) What is $\mathbb{P}[X \geq 1]$ ? Hint: You should be able to do this without a summation.
(d) What is $\mathbb{P}[12 \leq X \leq 14]$ ?

## 2 Head Count II

Consider a coin with $\mathbb{P}[$ Heads $]=3 / 4$. Suppose you flip the coin until you see heads for the first time, and define $X$ to be the number of times you flipped the coin.
(a) What is $\mathbb{P}[X=k]$, for some $k \geq 1$ ?
(b) Name the distribution of $X$ and what its parameters are.
(c) What is $\mathbb{P}[X>k]$, for some $k \geq 0$ ?
(d) What is $\mathbb{P}[X<k]$, for some $k \geq 1$ ?
(e) What is $\mathbb{P}[X>k \mid X>m]$, for some $k \geq m \geq 0$ ? How does this relate to $\mathbb{P}[X>k-m]$ ?
(f) Suppose $X \sim \operatorname{Geometric}(p)$ and $Y \sim \operatorname{Geometric}(q)$ are independent. Find the distribution of $\min (X, Y)$ and justify your answer.

## 3 Shuttles and Taxis at Airport

In front of terminal 3 at San Francisco Airport is a pickup area where shuttles and taxis arrive according to a Poisson distribution. The shuttles arrive at a rate $\lambda_{1}=1 / 20$ (i.e. 1 shuttle per 20 minutes) and the taxis arrive at a rate $\lambda_{2}=1 / 10$ (i.e. 1 taxi per 10 minutes) starting at 00:00. The shuttles and the taxis arrive independently.
(a) What is the distribution of the following:
(i) The number of taxis that arrive between times 00:00 and 00:20?
(ii) The number of shuttles that arrive between times 00:00 and 00:20?
(iii) The total number of pickup vehicles that arrive between times 00:00 and 00:20?
(b) What is the probability that exactly 1 shuttle and 3 taxis arrive between times 00:00 and 00:20?
(c) Given that exactly 1 pickup vehicle arrived between times 00:00 and 00:20, what is the conditional probability that this vehicle was a taxi?
(d) Suppose you reach the pickup area at 00:20. You learn that you missed 3 taxis and 1 shuttle in those 20 minutes. What is the probability that you need to wait for more than 10 mins until either a shuttle or a taxi arrives?

