CS 70 Fall 2023

Discrete Mathematics and Probability Theory Tal, Rao

DIS 9B

1 Head Count

Note 15

Consider a coin with $\mathbb{P}[\text{Heads}] = 2/5$. Suppose you flip the coin 20 times, and define X to be the number of heads.

(a) What is $\mathbb{P}[X = k]$, for some $0 \le k \le 20$?

(b) Name the distribution of X and what its parameters are.

(c) What is $\mathbb{P}[X \ge 1]$? Hint: You should be able to do this without a summation.

(d) What is $\mathbb{P}[12 \le X \le 14]$?

2 Head Count II

Consider a coin with $\mathbb{P}[\text{Heads}] = 3/4$. Suppose you flip the coin until you see heads for the first time, and define *X* to be the number of times you flipped the coin.

(a) What is $\mathbb{P}[X = k]$, for some $k \ge 1$?

(b) Name the distribution of *X* and what its parameters are.

- (c) What is $\mathbb{P}[X > k]$, for some $k \ge 0$?
- (d) What is $\mathbb{P}[X < k]$, for some $k \ge 1$?

(e) What is $\mathbb{P}[X > k \mid X > m]$, for some $k \ge m \ge 0$? How does this relate to $\mathbb{P}[X > k - m]$?

(f) Suppose $X \sim \text{Geometric}(p)$ and $Y \sim \text{Geometric}(q)$ are independent. Find the distribution of $\min(X,Y)$ and justify your answer.

3 Shuttles and Taxis at Airport

Note 19

In front of terminal 3 at San Francisco Airport is a pickup area where shuttles and taxis arrive according to a Poisson distribution. The shuttles arrive at a rate $\lambda_1 = 1/20$ (i.e. 1 shuttle per 20 minutes) and the taxis arrive at a rate $\lambda_2 = 1/10$ (i.e. 1 taxi per 10 minutes) starting at 00:00. The shuttles and the taxis arrive independently.

- independently.

 (a) What is the distribution of the following:

 (i) The number of taxis that arrive between times 00:00 and 00:20?

 (ii) The number of shuttles that arrive between times 00:00 and 00:20?

 (iii) The total number of pickup vehicles that arrive between times 00:00 and 00:20?

 (b) What is the probability that exactly 1 shuttle and 3 taxis arrive between times 00:00 and 00:20?

 (c) Given that exactly 1 pickup vehicle arrived between times 00:00 and 00:20, what is the conditional probability that this vehicle was a taxi?
- (d) Suppose you reach the pickup area at 00:20. You learn that you missed 3 taxis and 1 shuttle in those 20 minutes. What is the probability that you need to wait for more than 10 mins until either a shuttle or a taxi arrives?

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