

1 Student Life

Note 19

In an attempt to avoid having to do laundry often, Marcus comes up with a system. Every night, he designates one of his shirts as his dirtiest shirt. In the morning, he randomly picks one of his shirts to wear. If he picked the dirtiest one, he puts it in a dirty pile at the end of the day (a shirt in the dirty pile is not used again until it is cleaned).

When Marcus puts his last shirt into the dirty pile, he finally does his laundry, and again designates one of his shirts as his dirtiest shirt (laundry isn't perfect) before going to bed. This process then repeats.

(a) If Marcus has n shirts, what is the expected number of days that transpire between laundry events? Your answer should be a function of n involving no summations.

(b) Say he gets even lazier, and instead of organizing his shirts in his dresser every night, he throws his shirts randomly onto one of n different locations in his room (one shirt per location), designates one of his shirts as his dirtiest shirt, and one location as the dirtiest location.

In the morning, if he happens to pick the dirtiest shirt, *and* the dirtiest shirt was in the dirtiest location, then he puts the shirt into the dirty pile at the end of the day and does not throw any future shirts into that location and also does not consider it as a candidate for future dirtiest locations (it is too dirty).

What is the expected number of days that transpire between laundry events now? Again, your answer should be a function of n involving no summations.

2 Such High Expectations

Note 19

Suppose X and Y are independently drawn from a Geometric distribution with parameter p .

(a) Compute $\mathbb{E}[\min(X, Y)]$.

(b) Compute $\mathbb{E}[\max(X, Y)]$.

3 Dice Variance

Note 16

(a) Let X be a random variable representing the outcome of the roll of one fair 6-sided die. What is $\text{Var}(X)$?

(b) Let Z be a random variable representing the average of n rolls of a fair 6-sided die. What is $\text{Var}(Z)$?

4 Covariance

Note 16

- (a) We have a bag of 5 red and 5 blue balls. We take two balls uniformly at random from the bag without replacement. Let X_1 and X_2 be indicator random variables for the events of the first and second ball being red, respectively. What is $\text{cov}(X_1, X_2)$? Recall that $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.
- (b) Now, we have two bags A and B, with 5 red and 5 blue balls each. Draw a ball uniformly at random from A, record its color, and then place it in B. Then draw a ball uniformly at random from B and record its color. Let X_1 and X_2 be indicator random variables for the events of the first and second draws being red, respectively. What is $\text{cov}(X_1, X_2)$?