1 Elevator Variance

Note 16

A building has n upper floors numbered $1,2,\ldots,n$, plus a ground floor G. At the ground floor, m people get on the elevator together, and each person gets off at one of the n upper floors uniformly at random and independently of everyone else. What is the *variance* of the number of floors the elevator *does not* stop at?

2 Covariance

Note 16

(a) We have a bag of 5 red and 5 blue balls. We take two balls uniformly at random from the bag without replacement. Let X_1 and X_2 be indicator random variables for the events of the first and second ball being red, respectively. What is $\text{cov}(X_1,X_2)$? Recall that $\text{cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$.

(b) Now, we have two bags A and B, with 5 red and 5 blue balls each. Draw a ball uniformly at random from A, record its color, and then place it in B. Then draw a ball uniformly at random from B and record its color. Let X_1 and X_2 be indicator random variables for the events of the first and second draws being red, respectively. What is $\text{cov}(X_1, X_2)$?

3 Number Game

Note 20

Sinho and Vrettos are playing a game where they each choose an integer uniformly at random from $[0, 100]$, then whoever has the larger number wins (in the event of a tie, they replay). However, Vrettos doesn't like losing, so he's rigged his random number generator such that it instead picks randomly from the integers between Sinho's number and 100. Let S be Sinho's number and V be Vrettos' number.

(a) What is $\mathbb{E}[S]$?

(b) What is $\mathbb{E}[V \mid S = s]$, where s is any constant such that $0 \le s \le 100$?

(c) What is $\mathbb{E}[V]$?