70: Discrete Math and Probability Theory

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Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
Logic and Proofs!
Induction $\equiv$ Recursion.

What can computers do?
Work with discrete objects.
Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...
Probability!

The truth: My hopes and dreams.

You learn to think more clearly and more powerfully.

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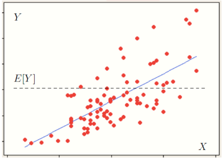
You learn to think more clearly and more powerfully.

..And to deal clearly with uncertainty itself.

Probability Unit

- How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, …)?
  - Constructive Models: Model the overall system (including the sources of uncertainty).
    - For modeling uncertainty, we'll develop probabilistic models and techniques for analyzing them.
  - Deductive Models: Extract the "trend" from the previous outcomes (e.g., linear regression).



Learning.

Veritassium on Khan

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Confusion is the sweat of learning.

Learning.

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Confusion is the sweat of learning.

Confusion is the sweat of discovery.

Metacogition.

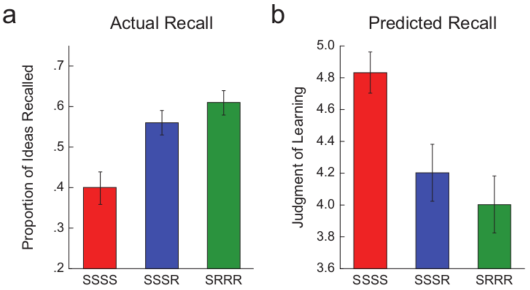

Fig. 1. Final recall (a) after repeatedly studying a text in four study periods (SSSS condition), reading a text in three study periods and then recalling it in one retrieval period (SSSR condition), or reading a text in one study period and then repeatedly recalling it in three retrieval periods (SRRR condition). Judgments of learning (b) were made on a 7-point scale, where 7 indicated that students believed they would remember material very well. The data presented in these graphs are adapted from Experiment 2 of Roediger and Karpicke (2006b). The pattern of students' metacognitive judgments of learning (predicted recall) was exactly the opposite of the pattern of students' actual long-term retention.

Learning styles.

How to search google.

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"Learning styles"

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How to search google.
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CS70: Notes, lectures, discussions, vitamins, homeworks.

An effective student is...

Smart, rich,

An effective student is...

Smart, rich,

and beautiful.

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All memes. The last one is not a meme.
First one, learning is inherent.

An effective student is...

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All memes. The last one is not a meme.
First one, learning is inherent. You are all capable.
Second, background, background, etc.

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First one, learning is inherent. You are all capable.

Second, background, background, etc. The material is doable.

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What I think.

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What I think.

- Confident, motivated,

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What I think.

Confident, motivated,

has integrity.

Known knowns..

There are the known knowns, known unknowns, and unknown unknowns.

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The last one is what always gets you.

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Its being comfortable with what you don’t know.
In order to get there.

Dogs don’t have rights cuz..

Dogs don't have rights cuz..

They don't know infinity.

First grade

$1, 2, 3, 4, \ldots, 120$

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Peano's axioms. There is always a successor.

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$+3$ means move to successor and another and another, or 3 times.

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Metric. (distance.) There is mapping $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$
Obeys triangle inequality: $f(i, j) + f(j, k) \geq f(i, k)$

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11 is one ten, and one one.

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Computer science: efficient representation of a number.
Algorithms: how to add.
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3×5?

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3×5?

$\times$ means add 3 times.
$5 + 5 + 5$
10 is moving over 5 from 5
The next number one can use the one's place.

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Notes cover material. Discussion. Vitamins. Homework. Study.

How to interact with staff..

My advice to TA's.

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When a student asks questions, probe to see where they are.

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When a student asks questions, probe to see where they are. And then move them forward.

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What should you do?

How to interact with staff..

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E.g., Avoid long explanations with nodding students. You must checkin meaningfully.

What should you do?
Where does your understanding get iffy?

How to interact with staff..

My advice to TA's.

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E.g., Avoid long explanations with nodding students. You must checkin meaningfully.

What should you do?
Where does your understanding get iffy?
Explain what you understand, then say what you don't.

Advice from (former) TA's

Dinstiguished Almunus (DA) Megan:

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I read the notes until I could reproduce the proofs myself.

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Head TA Richard:
"carefully review the homework solutions after they are released and understand them to the point of being able to replicate them without needing to reference them."

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Mini-vitamins.

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   Do before lecture.

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   But, it’s before it’s taught!

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Please do not take it out on your TA's.

Admin

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Explains policies, has office hours, homework, midterm dates, etc.

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One midterm, final.

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Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

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Questions

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Announcements, logistics, critical advice.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

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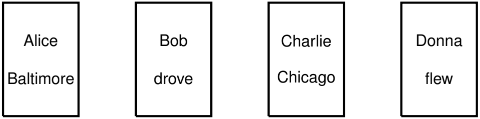
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.



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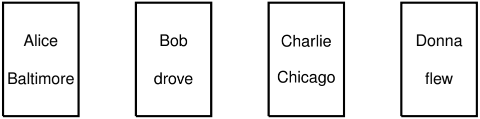

- Which cards must you flip to test the theory?

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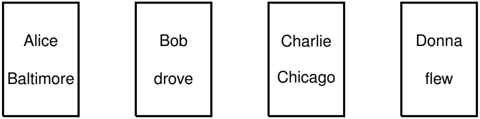

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Answer: (A), (B), (C), (D).

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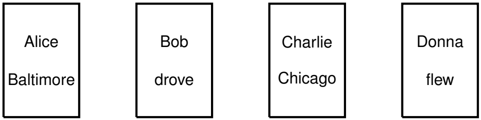

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Answer: (A), (B), (C), (D). Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
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Any even > 2 is sum of 2 primes
$4 + 5$
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Propositional Forms.

Put propositions together to make another...

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Put them together..

Propositions:

P_1 - Person 1 rides the bus.

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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

....

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This seems ...complicated.

We can program!!!!

We need a way to keep track!

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True if
both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	
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Check: $\wedge$ and $\vee$ are commutative.

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
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$$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q$$

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One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
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T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q \qquad\qquad \neg(P \vee Q)$

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Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q \qquad \neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
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Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
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Quick Questions

P	Q	$P \wedge Q$
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Is $(T \wedge Q) \equiv Q$? Yes?

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P	Q	$P \vee Q$
T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

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What is $(F \wedge Q)$? F or False.

Quick Questions

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What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$?

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T	T	T
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What is $(F \wedge Q)$? F or False.

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Quick Questions

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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$? Q

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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P is False .

LHS: $F \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

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LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

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Simplify: $T \vee Q \equiv T$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
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LHS: $F \wedge (Q \vee R) \equiv F$.
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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:
$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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True Statements: P, $P \implies Q$.

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Implication.

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Examples:

Statement: If you stand in the rain, then you'll get wet.

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True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

Implication.

$P \implies Q$ interpreted as

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True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
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$P \implies Q$ interpreted as

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Examples:

Statement: If you stand in the rain, then you'll get wet.
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Statement: "Stand in the rain"

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$P \implies Q$ interpreted as

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Statement: If you stand in the rain, then you'll get wet.
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Statement: "Stand in the rain"
Can conclude: "you'll get wet."

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Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

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P = "a right triangle has sidelengths $a \leq b \leq c$",
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Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

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P False means Q can be True or False

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If chemical plant pollutes river, fish die.

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If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

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Not necessarily.

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The chemical plant pollutes river. Can we conclude fish die?

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The chemical plant pollutes river. Can we conclude fish die?

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.

  Just reversing the order.

Implication and English.

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Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.

Implication and English.

$P \implies Q$

Poll.

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- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.

Implication and English.

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  Remember if P is true then Q must be true.

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  since if Q is false P must have been false.
- P is sufficient for Q.

Implication and English.

$P \implies Q$

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  Just reversing the order.
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Implication and English.

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  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.

Implication and English.

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  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.
  Example: It is necessary that $n > 3$ for $n > 4$.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
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P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
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P	Q	$\neg P \vee Q$
T	T	
T	F	
F	T	
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Truth Table: implication.

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T	T	T
T	F	F
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Truth Table: implication.

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$\neg P \vee Q \quad \equiv P \implies Q$.

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$\neg P \vee Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
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- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)

  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.

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- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
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- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
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  Logically equivalent! Notation: $\equiv$.

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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
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Next: Statements about boolean valued functions!!

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- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
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Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

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$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x)$

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Statement/theory: $\forall x \in \{A, B, C, D\}, \mathit{Chicago}(x) \implies \mathit{Flew}(x)$

$\mathit{Chicago}(A)$ = False .

Back to: Wason's experiment:1

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So $Chicago(Bob)$ must be False .

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$Chicago(C)$ = True .

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$Flew(D)$ = True . Do we care about $Chicago(D)$?
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Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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$$(\forall x \in N)\,(2x > x)$$ **False**

More for all quantifiers examples.

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$$(\forall x \in N)\ (2x > x) \quad \textbf{False} \quad \textbf{Consider } x = 0$$

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  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

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- "Square of any natural number greater than 5 is greater than 25."

More for all quantifiers examples.

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- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

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  Idea alert:

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  Idea alert: Restrict domain using implication.

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Later we may omit universe if clear from context.

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

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Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

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What we do in this course! We consider claims.

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Claim: $(\forall x)\ P(x)$

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Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.

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Claim: $(\forall x)\ P(x)$ “For all inputs x the program works.”
For False , find x, where $\neg P(x)$.
Counterexample.

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Next Time: proofs!