Today.

Last time:

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Shared (and sort of kept) secrets.

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Today: Errors

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  Tolerate Loss: erasure codes.

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Given points: $(x_1, y_1); (x_2, y_2) \cdots (x_k, y_k)$.

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\vdots & \vdots & \vdots & \\
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Modular Arithmetic Fact: Exactly 1 polynomial of degree $\leq d$ with arithmetic modulo prime p contains $d+1$ pts.

Proof sketches.

Property 2 A polynomial: $P(x) = a_d x^d + \cdots a_0$ has $d+1$ coefficients.

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Multiply by zero. My love is won.
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Property 1 A non-zero degree d polynomial has at most d roots.

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Love me some contradiction!
Two polynomials: $P(x), Q(x)$, $P(x) - Q(x)$ has too many roots.

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Proof works for reals, rationals, and complex numbers.

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In the rationals, the precision blows up, where in modular arithmetic, it does not.

Secret Sharing

Modular Arithmetic Fact: Exactly one polynomial degree $\leq d$ over $GF(p)$, $P(x)$, that hits $d+1$ points.

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Knowing k pts, only one $P(x)$, evaluate $P(0)$.
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3 kids hand out 3 points. Any two know the line.

Secret Sharing.

n people, k is enough.

(A) The modulus needs to be at least $n+1$.
(B) The modulus needs to be at least k.
(C) Use degree k polynomial, hand out n points.
(D) Use degree n polynomial, hand out k points.
(E) Use degree $k-1$ polynomial, hand out n points.
(F) The modulus needs to be at least 2^s, where s is value of secret.
(G) The modulus needs to be at least 2^s, where s is size of secret.

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(A), (B), (E), (F)

Erasure Codes.

Satellite

GPS device

Erasure Codes.

Satellite

3 packet message.

GPS device

Erasure Codes.

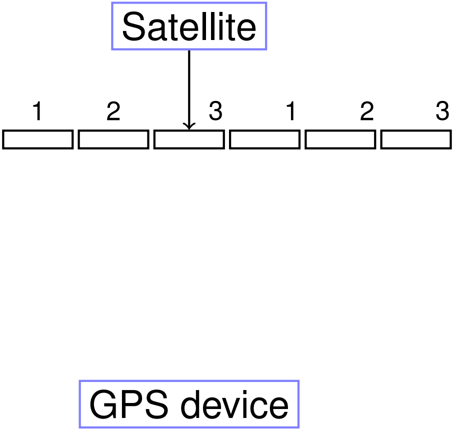
Satellite

3 packet message.

Lose 3 out 6 packets.

GPS device

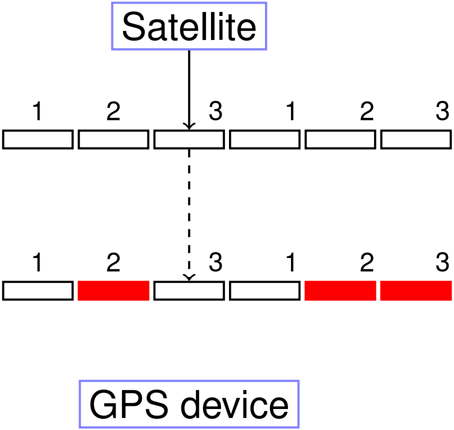
Erasure Codes.



3 packet message. So send 6!

Lose 3 out 6 packets.

Erasure Codes.



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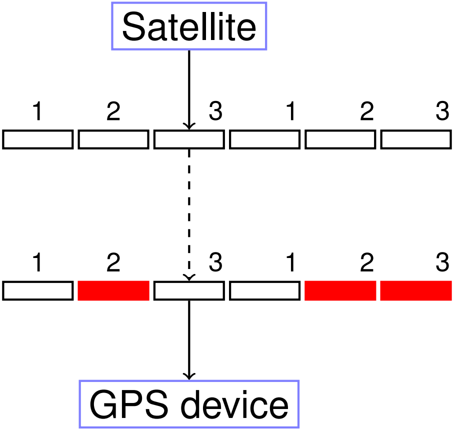

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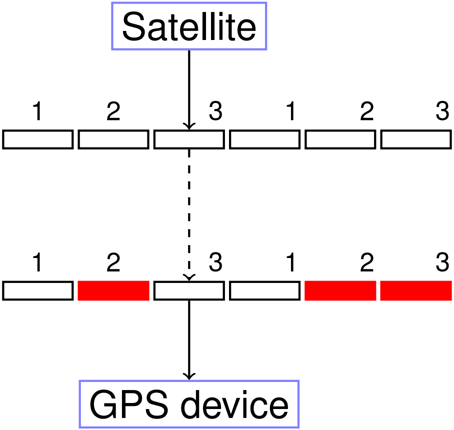

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Gets packets 1,1,and 3.

Solution Idea.

n packet message, channel that loses k packets.

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Use polynomials.

The Scheme

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A degree $n-1$ polynomial determined by any n points!

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GPS device

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Satellite

n packet message.

GPS device

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Satellite

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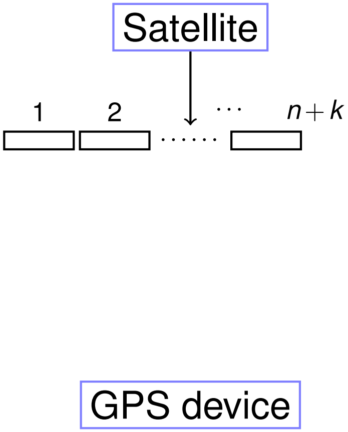
Lose *k* packets.

GPS device

Erasure Codes.



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Lose k packets.

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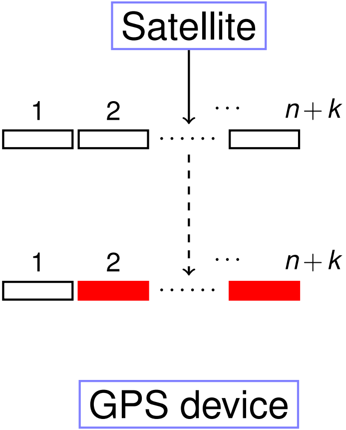

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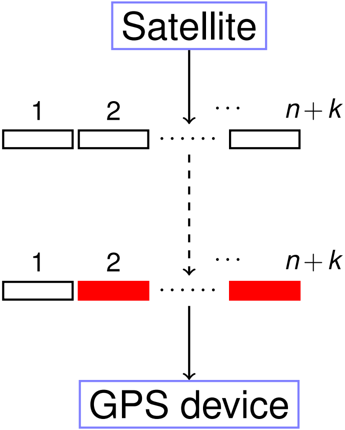

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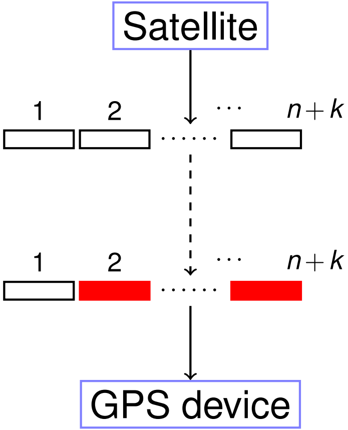

n packet message. So send $n+k$!

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Any n packets is enough!

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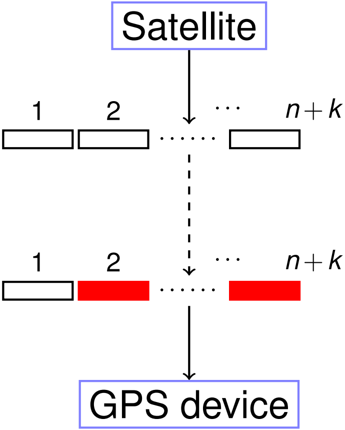

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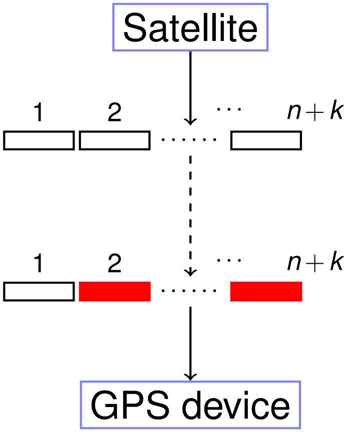

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Optimal.

Information Theory.

Size: Can choose a prime between 2^{b-1} and 2^b.
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$P(x) = 2x^2 + 4x + 2$

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Packets: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

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Packets: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

Notice that packets contain “x-values”.

Bad reception!

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Channeling Sahai

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Recieve: (1,1) (2,4), (6,0)
Reconstruct?

Format: $(i, R(i))$.

Lagrange or linear equations.

$$\begin{aligned}
P(1) = a_2 + a_1 + a_0 &\equiv 1 \pmod 7 \\
P(2) = 4a_2 + 2a_1 + a_0 &\equiv 4 \pmod 7 \\
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Channeling Sahai ...

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Message? $P(1) = 1,$

Bad reception!

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Message? $P(1) = 1, P(2) = 4, P(3) = 4$.

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You want to encode a secret consisting of 1,4,4.

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through a noisy channel that loses 3 packets.

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The other constraint: arithmetic system can represent $0,1,2,3,4$.

Send n packets b-bit packets, with k errors.
Modulus should be larger than $n+k$ and also larger than 2^b.

Polynomials.

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- ..give Secret Sharing.

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Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Error Correction

Satellite

3 packet message.

GPS device

Error Correction

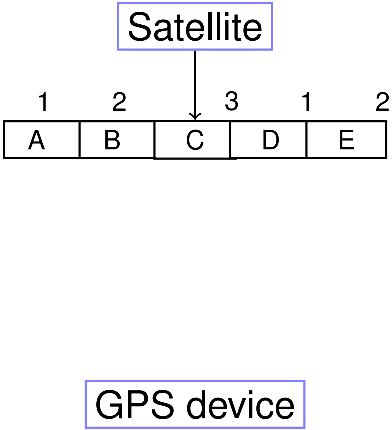
Satellite

3 packet message.

Corrupts 1 packets.

GPS device

Error Correction



3 packet message. Send 5.

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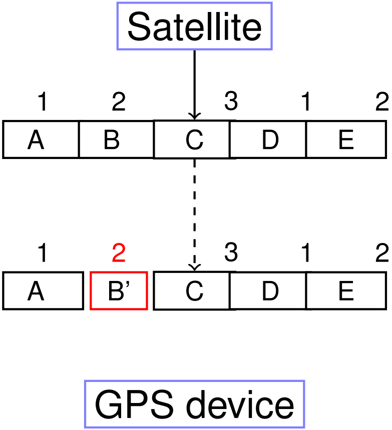

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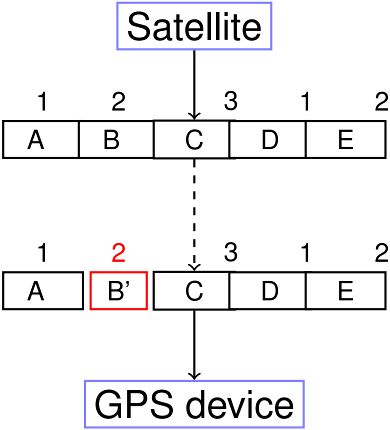

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$P(x)=R(i)$, on set of size $n+k$.

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Example.

Message: $3, 0, 6$.

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Reed Solomon Code: $P(x) = x^2 + x + 1 \pmod 7$ has $P(1) = 3, P(2) = 0, P(3) = 6$ modulo 7.

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(Aside: Message in plain text!)

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(Aside: Message in plain text!)

Receive $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$.

Example.

Message: $3, 0, 6$.

Reed Solomon Code: $P(x) = x^2 + x + 1 \pmod 7$ has $P(1) = 3, P(2) = 0, P(3) = 6$ modulo 7.

Send: $P(1) = 3, P(2) = 0, P(3) = 6, P(4) = 0, P(5) = 3$.

(Aside: Message in plain text!)

Receive $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$.

$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:
For each subset of $n+k$ points

Slow solution.

Brute Force:
For each subset of $n+k$ points
  Fit degree $n-1$ polynomial, $Q(x)$, to n of them.

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For each subset of $n+k$ points
  Fit degree $n-1$ polynomial, $Q(x)$, to n of them.
  Check if consistent with $n+k$ of the total points.

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Brute Force:

For each subset of $n+k$ points

  Fit degree $n-1$ polynomial, $Q(x)$, to n of them.

  Check if consistent with $n+k$ of the total points.

  If yes, output $Q(x)$.

Slow solution.

Brute Force:
For each subset of $n+k$ points
Fit degree $n-1$ polynomial, $Q(x)$, to n of them.
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- For subset of $n+k$ pts where $R(i) = P(i)$, method will reconstruct $P(x)$!

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- For subset of $n+k$ pts where $R(i) = P(i)$, method will reconstruct $P(x)$!
- For any subset of $n+k$ pts,
    1. unique degree $n-1$ polynomial $Q(x)$ that fits $\geq n$ of them

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- For any subset of $n+k$ pts,
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  2. and where $Q(x)$ is consistent with $n+k$ points

Slow solution.

Brute Force:
For each subset of $n+k$ points
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  $\Longrightarrow P(x) = Q(x)$.

Slow solution.

Brute Force:
For each subset of $n+k$ points
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- For subset of $n+k$ pts where $R(i) = P(i)$, method will reconstruct $P(x)$!
- For any subset of $n+k$ pts,
  1. unique degree $n-1$ polynomial $Q(x)$ that fits $\geq n$ of them
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  $\implies P(x) = Q(x)$.

Reconstructs $P(x)$ and only $P(x)$!!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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Find $P(x)=p_2x^2+p_1x+p_0$ that contains $n+k=3+1$ points.

All equations..

$$\begin{aligned}
p_2+p_1+p_0 &\equiv 3 \pmod 7\\
4p_2+2p_1+p_0 &\equiv 1 \pmod 7\\
2p_2+3p_1+p_0 &\equiv 6 \pmod 7\\
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Assume point 1 is wrong

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Assume point 1 is wrong and solve..

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Assume point 1 is wrong and solve..no consistent solution!
Assume point 2 is wrong

Example.

Received $R(1)=3, R(2)=1, R(3)=6, R(4)=0, R(5)=3$

Find $P(x)=p_2x^2+p_1x+p_0$ that contains $n+k=3+1$ points.

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In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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Error!!

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Error!! .... Where???

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Error!! .... Where???
Could be anywhere!!!

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Error!! .... Where???
Could be anywhere!!! ...so try everywhere.

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Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

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Something like $(n/k)^k$ ...Exponential in k!.

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Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!.

How do we find where the bad packets are efficiently?!?!?!

Ditty...

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
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Ditty...

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has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone..

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

Ditty...

Oh where, Oh where
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Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
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With the polynomial well put

Ditty...

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With the polynomial well put
But the channel a bit wrong

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And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong
Where, oh where do we look?

Where oh where can my bad packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}$$

Where oh where can my bad packets be?

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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

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Zero times anything is zero!!!!!

Where oh where can my bad packets be?

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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!! My love is won.

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But which equations should we multiply by 0?

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We will use a polynomial!!!

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All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know.

Where oh where can my bad packets be?

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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

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Error locator polynomial: $(x-2)$.

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Error locator polynomial: $(x - 2)$.

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4 unknowns (p_0, p_1, p_2 and e),

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4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

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(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i) \pmod{p} \\
&\vdots \\
(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m) \pmod{p}
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\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
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and linear in a_i and coefficients of $E(x)$!

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$$Q(x) = a_{n+k-1}x^{n+k-1} + a_{n+k-2}x^{n+k-2} + \cdots a_0$$

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Finding $Q(x)$ and $E(x)$?

- $E(x)$ has degree k ...

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Number of unknown coefficients:

Finding $Q(x)$ and $E(x)$?

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Number of unknown coefficients: $n+2k$.

Solving for $Q(x)$ and $E(x)$...

For all points $1, \ldots, i, n+2k = m$,

$$Q(i) = R(i)E(i) \pmod{p}$$

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a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
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$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

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$Q(x) = x^3 + 6x^2 + 6x + 5.$

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$Q(x) = x^3 + 6x^2 + 6x + 5$.

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Example: finishing up.

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```
             ------------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
```

Example: finishing up.

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```
                 1 x^2 + 1 x
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
```

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```
                 1 x^2 + 1 x
         -----------------
 x - 2 ) x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
         ----------------
x - 2 )  x^3   + 6 x^2 + 6 x + 5
         x^3   - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
```

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$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x + 1
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

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```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2 + 1 x + 1
         -----------------
x - 2 )  x^3   + 6 x^2 + 6 x + 5
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         ----------
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                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                  1 x^2 + 6 x + 5
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                  ---------------
                            x + 5
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                            -----
                                0
```

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What is $\frac{x-2}{x-2}$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x + 1
          -----------------
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        x^3  - 2 x^2
        ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

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What is $\frac{x-2}{x-2}$? 1

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2 + 1 x + 1
          -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
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What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
            -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Proof:

Unique solution for $P(x)$

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Proof:
We claim

Unique solution for $P(x)$

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

Unique solution for $P(x)$

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Proof:
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$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

Equation 2 implies 1:

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Yaay!!

Berlekamp-Welsh algorithm decodes correctly when k errors!

Poll

Say you sent a message of length 4, encoded as $P(x)$ where one sends packets $P(1), ...P(8)$.

You recieve packets $R(1), ...R(8)$.

Packets 1 and 4 are corrupted.

(A) $R(1) \neq P(1)$
(B) The degree of $P(x)E(x) = 3+2 = 5$.
(C) The degree of $E(x)$ is 2.
(D) The number of coefficients of $P(x)$ is 4.
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(A), (C), (E). (F) doesn't type check!

Summary. Error Correction.

Communicate n packets, with k erasures.

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Reed-Solomon codes.

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Reed-Solomon codes. Welsh-Berlekamp Decoding.

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Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Cool.

Really Cool!