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Say you sent a message of length 4, encoded as $P(x)$ where one sends packets $P(1), ...P(8)$.

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Packets 1 and 4 are corrupted.

(A) $R(1) \neq P(1)$
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Reed-Solomon codes. Welsh-Berlekamp Decoding.

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
Why?
k changes to make diff. messages overlap
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
Recover?
Reconstruct error polynomial, $E(X)$, and $P(x)$!
Nonlinear equations.
Reconstruct $E(x)$ and $Q(x) = E(x)P(x)$. Linear Equations.
Polynomial division! $P(x) = Q(x)/E(x)$!

Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Cool.

Really Cool!

Probability

What's to come?

Probability

What's to come? Probability.

Probability

What's to come? Probability.

A bag contains:

Probability

What's to come? Probability.

A bag contains:

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Probability

What's to come? Probability.

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For now:

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

For now: Counting!

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

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For now: Counting!

Later: Probability.

The future in this course.

What's to come?

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What's to come? Probability.

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The future in this course.

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What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide. **Chances?**

The future in this course.

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide. **Chances?**

(A) Red Probability is $3/8$
(B) Blue probability is $3/9$
(C) Yellow Probability is $2/8$
(D) Blue probability is $3/8$

The future in this course.

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Today:

The future in this course.

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide. **Chances?**

(A) Red Probability is $3/8$
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Today: Counting!

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Probability is soon..but first let's count.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a n sided convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?
How many ways can I divide up 5 dollars among 3 people?

Using a tree..

How many 3-bit strings?

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How many different sequences of three bits from $\{0, 1\}$?

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How would you make one sequence?

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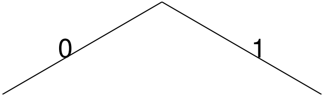

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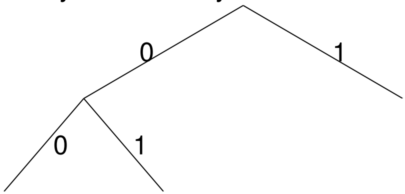

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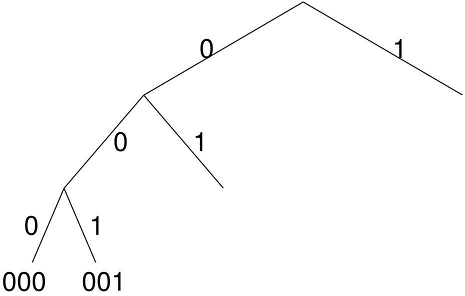

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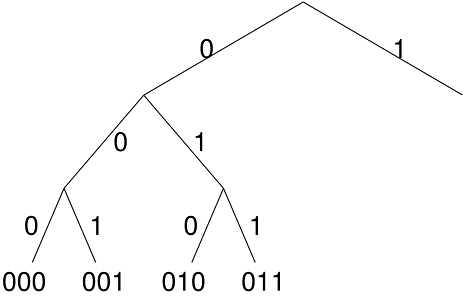

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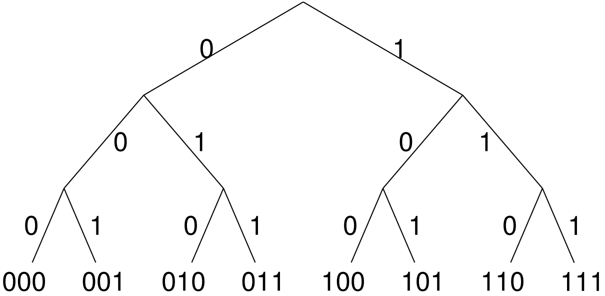

8 leaves which is $2 \times 2 \times 2$.

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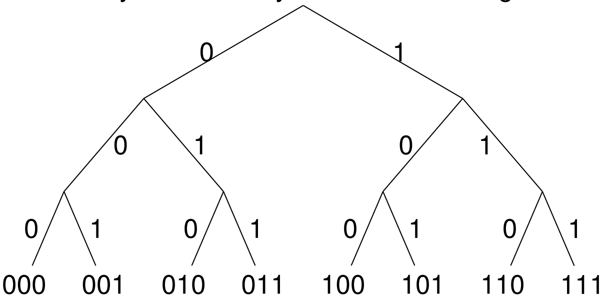

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

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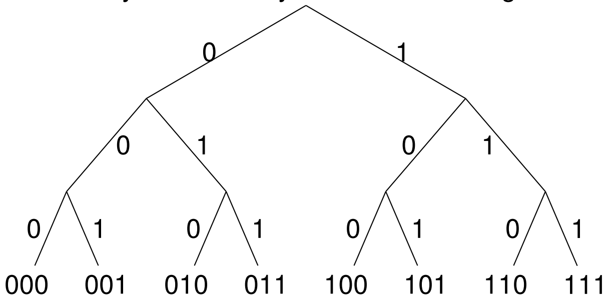

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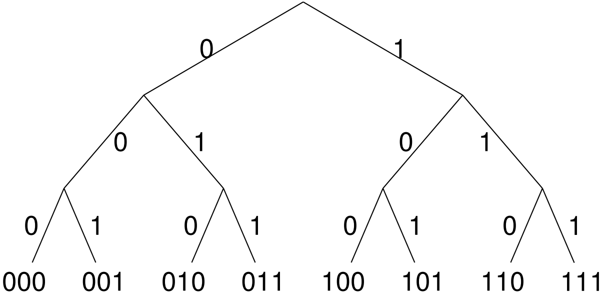

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8 3-bit strings!

First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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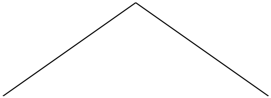

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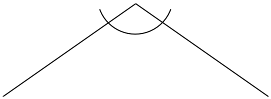

n_1

First Rule of Counting: Product Rule

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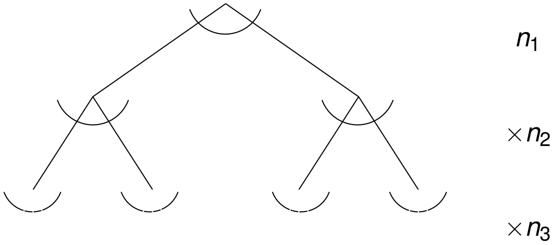

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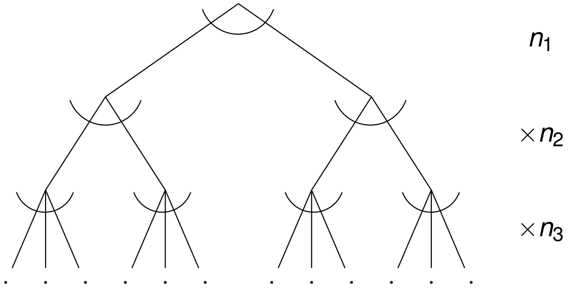

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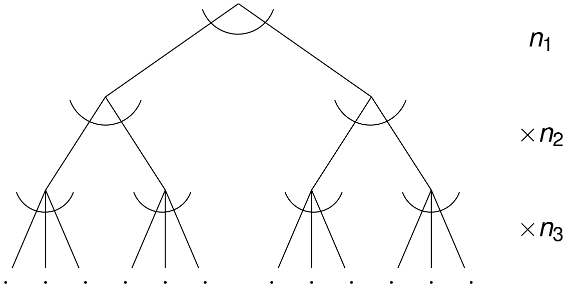

In picture, $2 \times 2 \times 3 = 12$!

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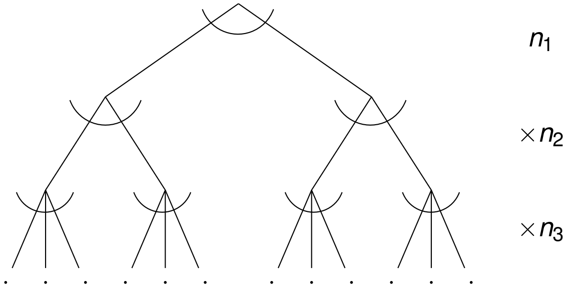

In picture, $2 \times 2 \times 3 = 12$!

Poll

Mark whats corect.

Poll

Mark whats corect.

(A) $|10 \text{ digit numbers}| = 10^{10}$
(B) $|k \text{ coin tosses}| = 2^k$
(C) $|10 \text{ digit numbers}| = 9 * 10^9$
(D) $|n \text{ digit base } m \text{ numbers}| = m^n$
(E) $|n \text{ digit base } m \text{ numbers}| = (m-1)m^{n-1}$

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(A) or (C)? (D) or (E)? (B) are correct.

Using the first rule..

How many outcomes possible for k coin tosses?

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How many outcomes possible for k coin tosses?

2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2×2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

Using the first rule..

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2 ways for first choice, 2 ways for second choice, ...
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How many 10 digit numbers?

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How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
10

Using the first rule..

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$2 \times 2 \cdots \times 2 = 2^k$

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$10 \times$

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(Is 09, a two digit number?)

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m^n

(Is 09, a two digit number?)

If no. Then $(m-1)m^{n-1}$.

Functions, polynomials.

How many functions f mapping S to T?

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$|T|$ ways to choose for $f(s_1)$,

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Questions?

Permutations.

[1] By definition: $0! = 1$.

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How many 10 digit numbers **without repeating a digit**?

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Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second,

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How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third,

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How many orderings of n objects are there?
Permutations of n objects.

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10 ways for first, 9 ways for second, 8 ways for third, ...

... $10 * 9 * 8 \cdots * 1 = 10!$.[1]

How many different samples of size k from n numbers **without replacement.**

n ways for first choice, $n-1$ ways for second,
$n-2$ choices for third, ...

... $n * (n-1) * (n-2) \cdot * (n-k+1) = \frac{n!}{(n-k)!}$.

How many orderings of n objects are there?
Permutations of n objects.

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[1] By definition: 0! = 1.

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One-to-One Functions.

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How many one-to-one functions from $|S|$ to $|S|$.

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

---

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

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$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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Are $A, K, Q, 10, J$ of spades
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Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

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Number of orderings for a poker hand: "5!"

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(The "!" means factorial, not Exclamation.)

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$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

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Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

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Number of orderings for a poker hand: "5!"

Can write as...
$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

[2] When each unordered object corresponds equal numbers of ordered objects.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

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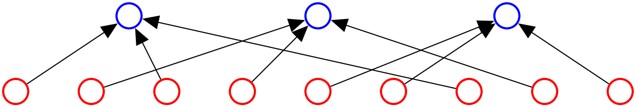

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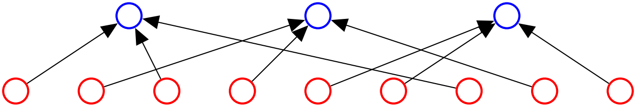

How many red nodes (ordered objects)?

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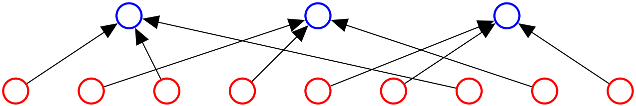

How many red nodes (ordered objects)? 9.

Ordered to unordered.

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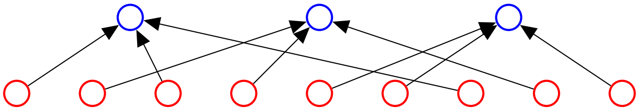
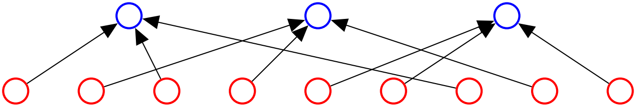


How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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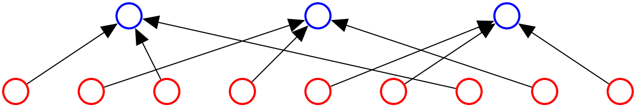

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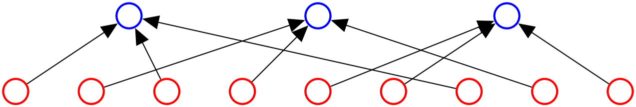

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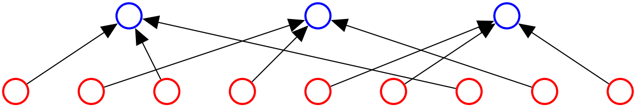

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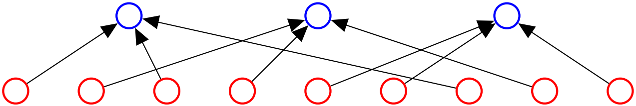

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How many poker deals?

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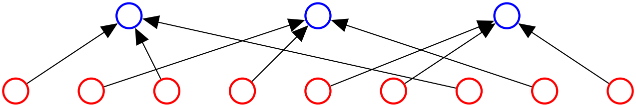

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How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

Ordered to unordered.

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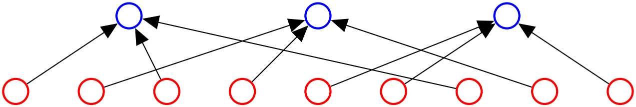

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How many poker deals per hand?

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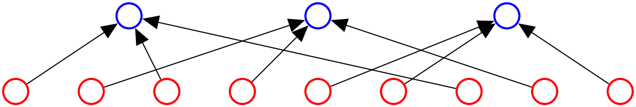
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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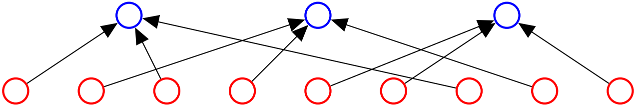
How many poker deals per hand?
Map each deal to ordered deal:

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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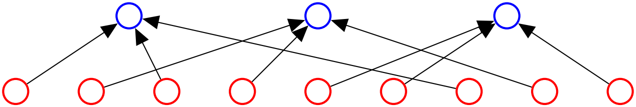
How many poker deals per hand?
Map each deal to ordered deal: $5!$

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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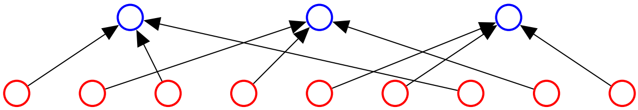

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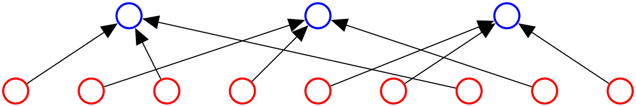

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Questions?

..order doesn’t matter.

..order doesn't matter.

Choose 2 out of n?

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{}$$

..order doesn't matter.

Choose 2 out of *n*?

$$\frac{n \times (n-1)}{2}$$

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Choose 3 out of n?

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$$\frac{n \times (n-1) \times (n-2)}{}$$

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Choose k **out of** n?

$$\frac{n!}{(n-k)!}$$

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Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

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Familiar?

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Choose 3 out of n?

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Choose k **out of** n?

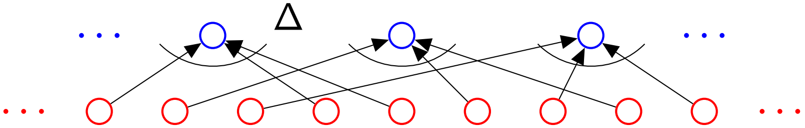
$$\frac{n!}{(n-k)! \times k!}$$

Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

Familiar? Questions?

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
Second rule: when order doesn't matter divide...



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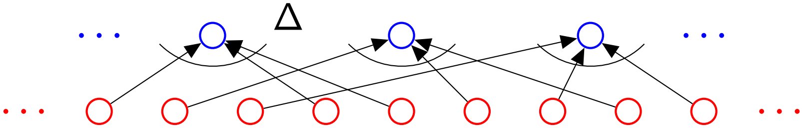
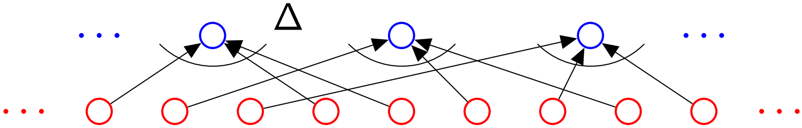


3 card Poker deals: 52

Example: Visualize the proof..

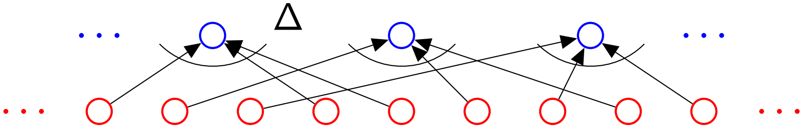
First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: 52×51

Example: Visualize the proof..

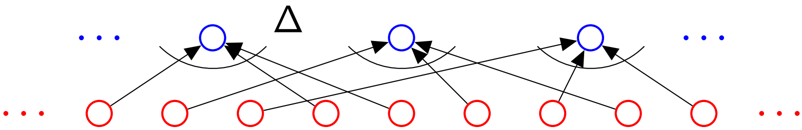
First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50$

Example: Visualize the proof..

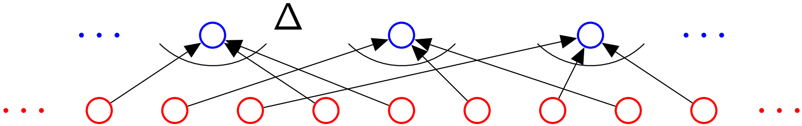
First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$.

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.

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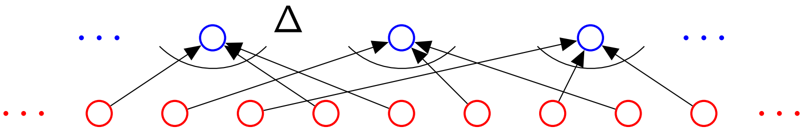
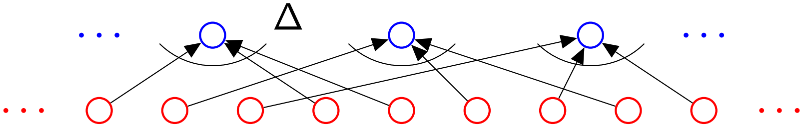


3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.

Example: Visualize the proof..

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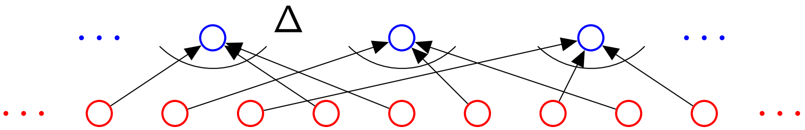

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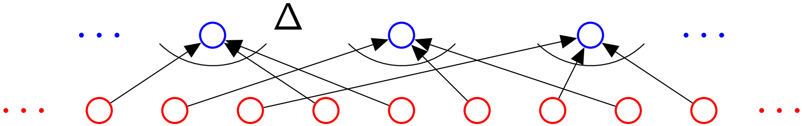

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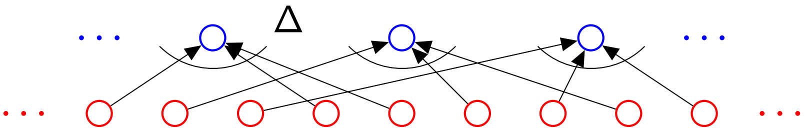

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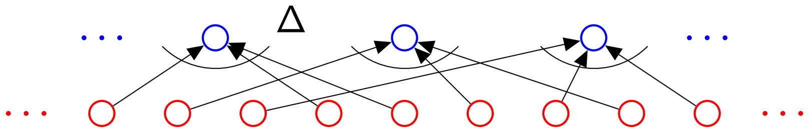

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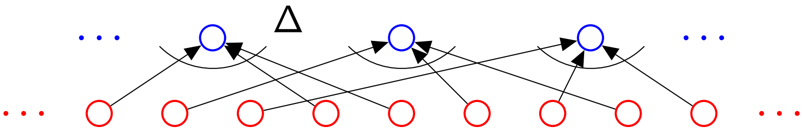

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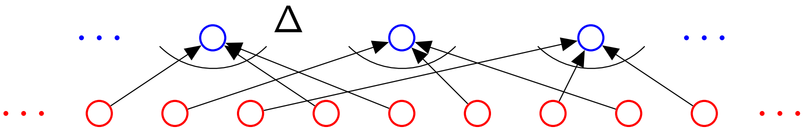

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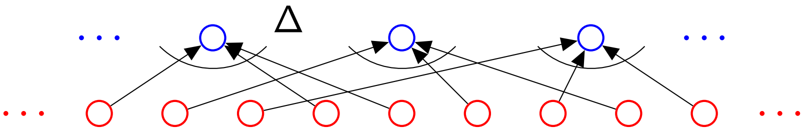

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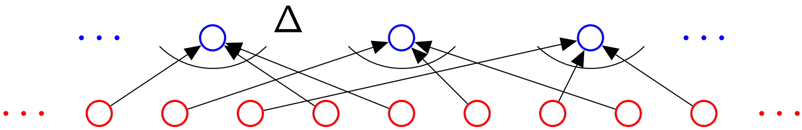

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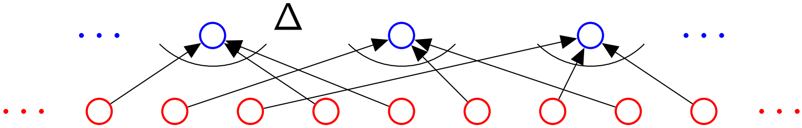

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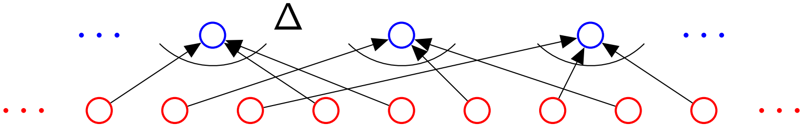

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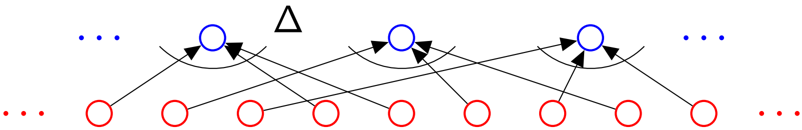

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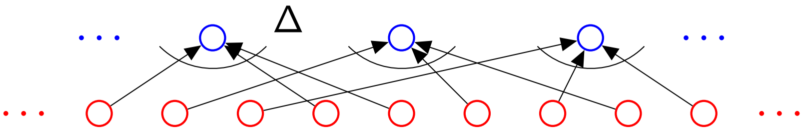

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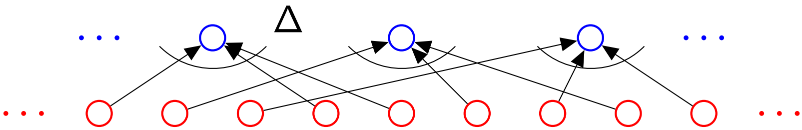

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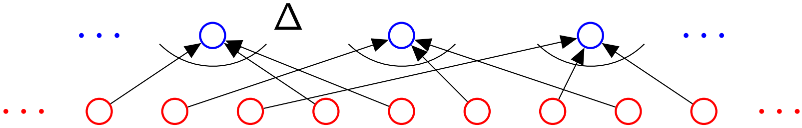

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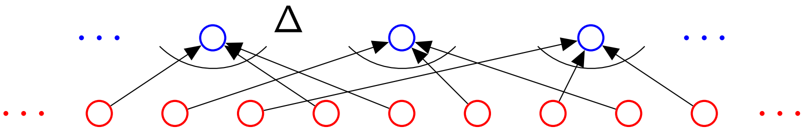

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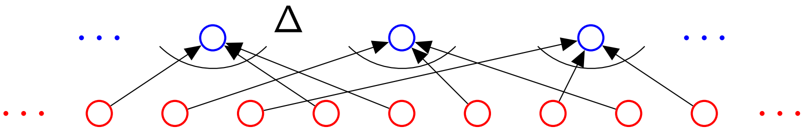

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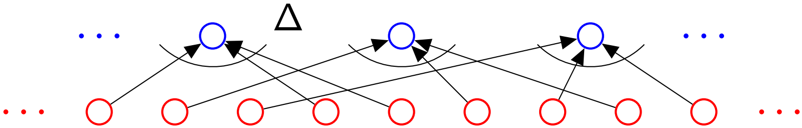

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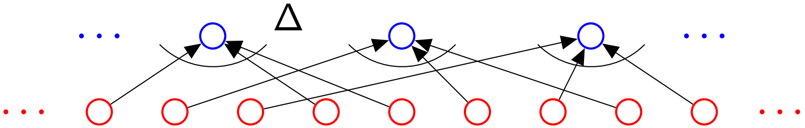

Orderings of ANAGRAM?

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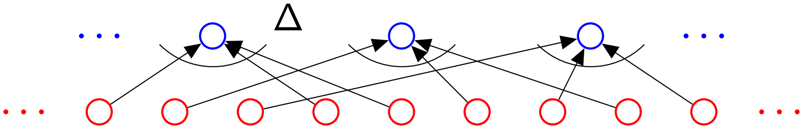

Orderings of ANAGRAM?
Ordered Set: 7!

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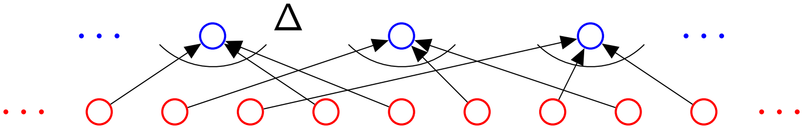

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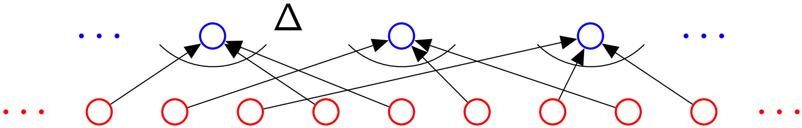

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A's are the same!

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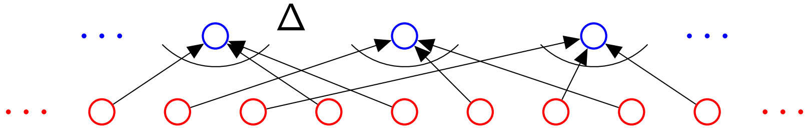

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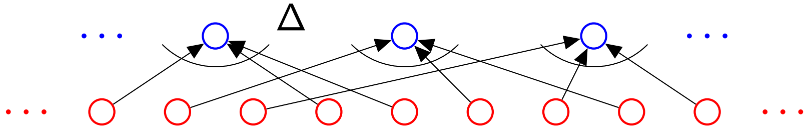

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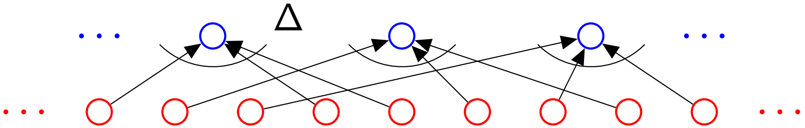

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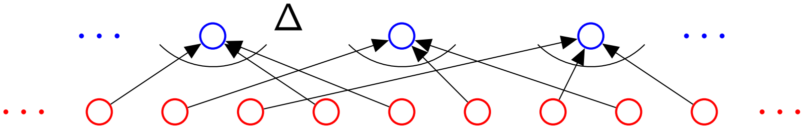

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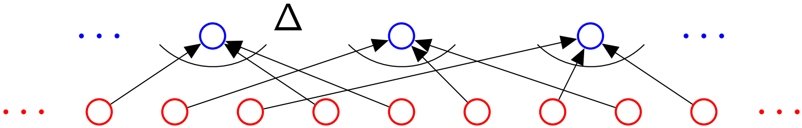

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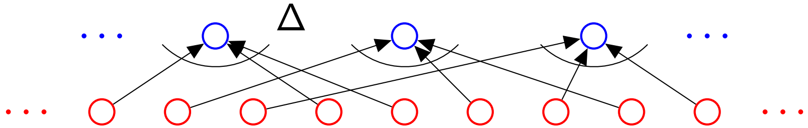

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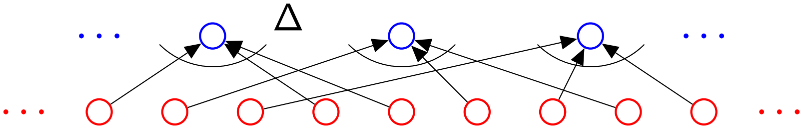

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$\Delta = 3 \times 2 \times 1 = 3!$

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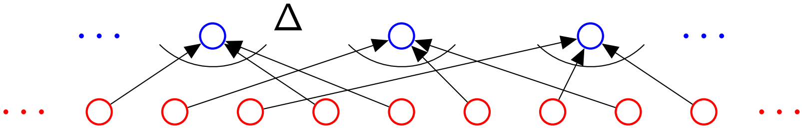

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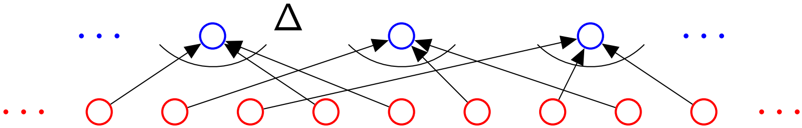

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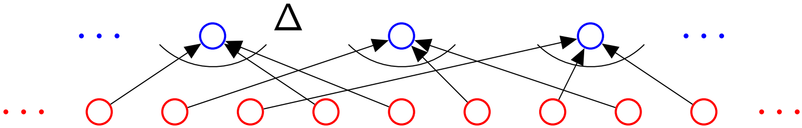

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Poll

Mark what’s correct.

Poll

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(A) $|\text{Poker hands}| = \binom{52}{5}$
(B) Orderings of ANAGRAM $= 7!/3!$
(C) Orderings of "CAT". $= 3!$
(D) Orders of MISSISSIPPI $= 11!/4!4!2!$
(E) Orderings of ANAGRAM $= 7!/4!$
(F) Orders of MISSISSIPPI $= 11!/10!$

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(F) Orders of MISSISSIPPI $= 11!/10!$

(A)-(E) are correct.

Some Practice.

How many orderings of letters of CAT?

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How many orderings of letters of CAT?

3 ways to choose first letter, 2 ways for second, 1 for last.

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How many orderings of the letters in ANAGRAM?

Some Practice.

How many orderings of letters of CAT?

3 ways to choose first letter, 2 ways for second, 1 for last.

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How many orderings of the letters in ANAGRAM?

Ordered,

Some Practice.

How many orderings of letters of CAT?

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How many orderings of the letters in ANAGRAM?

Ordered, except for A!

Some Practice.

How many orderings of letters of CAT?

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$\implies 3 \times 2 \times 1 = 3!$ orderings

How many orderings of the letters in ANAGRAM?

Ordered, except for A!

total orderings of 7 letters.

Some Practice.

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11 letters total.

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Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Sampling...

Sample k items out of n

Sampling...

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Without replacement:

Sampling...

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Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

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Order matters: $n \times n - 1 \times n - 2 \ldots$

Sampling...

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Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!}$

Order does not matter:

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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Second Rule: divide by number of orders

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With Replacement.

Order matters: $n \times n$

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Order matters: $n \times n \times \ldots n = n^k$

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Different number of unordered elts map to each unordered elt.

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Unordered elt: $1, 2, 3$

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Order does not matter:

Second Rule: divide by number of orders – “$k!$”

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Different number of unordered elts map to each unordered elt.

Unordered elt: $1, 2, 3$ $\quad$ 3! ordered elts map to it.

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Unordered elt: $1,2,2$ — $\frac{3!}{2!}$ ordered elts map to it.

How do we deal with this mess??

Splitting up some money....

How many ways can Bob and Alice split 5 dollars?

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For each of 5 dollars pick Bob or Alice(2^5), divide out order

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"Sorted" way to specify, first Alice's dollars, then Bob's.

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(A, B, B, B, B):
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and so on.

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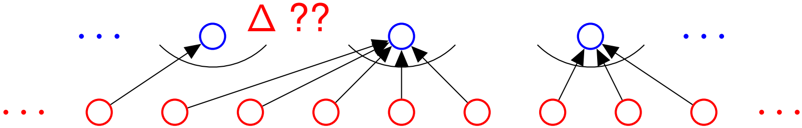
and so on.



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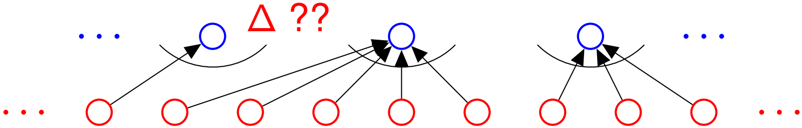

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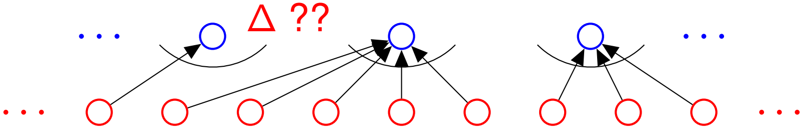
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.
(B, B, B, B, B): 1: (B,B,B,B,B)
(A, B, B, B, B): 5: (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
(A, A, B, B, B):
and so on.



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"Sorted" way to specify, first Alice's dollars, then Bob's.

(B,B,B,B,B): 1: (B,B,B,B,B)
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(A,A,B,B,B): $\binom{5}{2}$;(A,A,B,B,B),(A,B,A,B,B),(A,B,B,A,B),...

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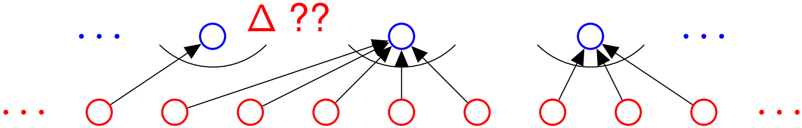

Second rule of counting is no good here!

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Separate Alice's dollars from Bob's and then Bob's from Eve's.

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Alice: 0, Bob: 1, Eve: 4.

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Each split "is" a sequence of stars and bars.

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Each split "is" a sequence of stars and bars.
Each sequence of stars and bars "is" a split.

Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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$| \star | \star \star \star \star$.

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7 positions in which to place the 2 bars.

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Stars and Bars.

How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

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How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

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Alice: 0; Bob 1; Eve: 4
$| \star | \star \star \star \star$.
Bars in first and third position.

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Bars in first and third position.

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How many different 5 star and 2 bar diagrams?

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7 positions in which to place the 2 bars.

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Bars in first and third position.

Alice: 1; Bob 4; Eve: 0
$\star | \star \star \star \star |$.
Bars in second and seventh position.

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$\binom{7}{2}$ ways to do so and

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7 positions in which to place the 2 bars.

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$| \star | \star \star \star \star$.
Bars in first and third position.

Alice: 1; Bob 4; Eve: 0
$\star | \star \star \star \star |$.
Bars in second and seventh position.

$\binom{7}{2}$ ways to do so and
$\binom{7}{2}$ ways to split 5 dollars among 3 people.

Stars and Bars.

Ways to add up n numbers to sum to k?

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Ways to add up n numbers to sum to k? or

" k from n with replacement where order doesn't matter."

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In general, k stars $n-1$ bars.

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$n+k-1$ positions from which to choose $n-1$ bar positions.

$$\binom{n+k-1}{n-1}$$

Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.

Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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k Samples with replacement from n items: n^k.

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Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$**.**

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Second rule: when order doesn't matter (sometimes) can divide...

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First rule: $n_1 \times n_2 \cdots \times n_3$.

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Sample without replacement and order doesn't matter: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.
"n choose k"

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Poll

Mark whats correct.

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Mark whats correct.

(A) ways to split k dollars among n: $\binom{k+n-1}{n-1}$

(B) ways to split n dollars among k: $\binom{n+k-1}{k-1}$

(C) ways to split 5 dollars among 3: $\binom{5+3-1}{3-1}$

(D) ways to split 5 dollars among 3: $\binom{7}{5}$

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All correct.

Quick review of the basics.

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Distribute k samples (stars) over n possibilities ($n-1$ bars group possibilities.)

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Sampling...

Sample k items out of n

Sampling...

Sample k items out of n

Without replacement:

Sampling...

Sample k items out of n

Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "k!"

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "$k!$"

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With Replacement.

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Order matters: n

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "$k!$"

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"n choose k"

With Replacement.

Order matters: $n \times n$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "k!"

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"n choose k"

With Replacement.

Order matters: $n \times n \times \ldots n$

Sampling...

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"n choose k"

With Replacement.

Order matters: $n \times n \times \ldots n = n^k$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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Order does not matter: Second rule

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Problem: depends on how many of each item we chose!

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Different number of unordered elts map to each unordered elt.

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Unordered elt: $1, 2, 3$

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Sample k items out of n

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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Unordered elt: $1, 2, 3$ $\quad$ 3! ordered elts map to it.

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Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "$k!$"

$\implies \frac{n!}{(n-k)!k!}$.

"n choose k"

With Replacement.

Order matters: $n \times n \times \ldots n = n^k$

Order does not matter: Second rule ???

Problem: depends on how many of each item we chose!

Different number of unordered elts map to each unordered elt.

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How many ways can Bob and Alice split 5 dollars?

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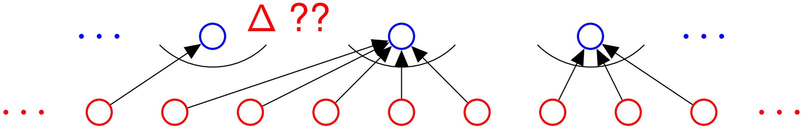

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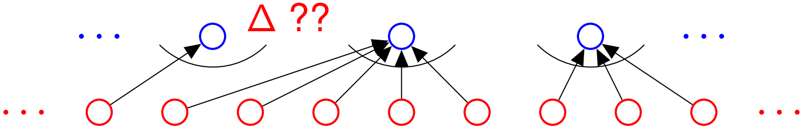

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(B, B, B, B, B): 1: (B,B,B,B,B)
(A, B, B, B, B): 5: (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
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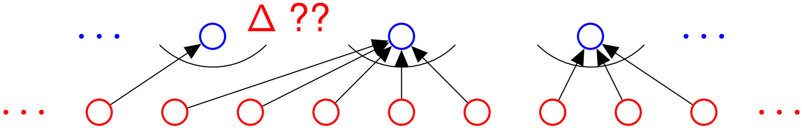

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(A, A, B, B, B): $\binom{5}{2}$;(A,A,B,B,B),(A,B,A,B,B),(A,B,B,A,B),...

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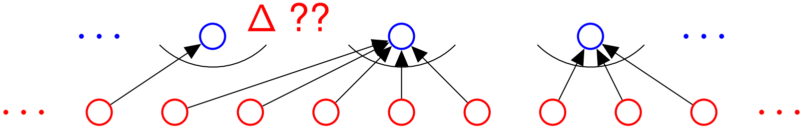

Second rule of counting is no good here!

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How many ways can Alice, Bob, and Eve split 5 dollars.

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Each split "is" a sequence of stars and bars.
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Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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$| \star | \star \star \star \star$.

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7 positions in which to place the 2 bars.

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Alice: 0; Bob 1; Eve: 4

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How many different 5 star and 2 bar diagrams?

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Alice: 0; Bob 1; Eve: 4
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Bars in first and third position.

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Bars in first and third position.

Alice: 1; Bob 4; Eve: 0

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How many different 5 star and 2 bar diagrams?

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Alice: 1; Bob 4; Eve: 0
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Bars in second and seventh position.

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Bars in second and seventh position.

$\binom{7}{2}$ ways to do so and
$\binom{7}{2}$ ways to split 5 dollars among 3 people.

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Ways to add up n numbers to sum to k?

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$$\binom{n+k-1}{n-1}$$

Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.