Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.

$\lim_{n \to \infty} (1 + 1/n)^n$.

$\sqrt{2}$? Algebraic number.

A solution of $x^2 = 2$.

Really, rationals seem fine for... say... calculus.

$\lim_{n \to \infty} \sum_{i=0}^{n} \frac{(b-a)}{n} f(x_i)$, where $x_i = a + i \times (b-a)/n$.

So why real numbers?

$\int_a^b f(x)dx$ is beautiful, succinct notation
        for a beautiful, succinct, powerful idea.

What's the idea? Area.

Generalized Continuum hypothesis.

There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

The powerset of a set is the set of all subsets.

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Can a program refer to a program?

Can a program refer to itself?

Uh oh....

The Barber!

The barber shaves every person who does not shave themselves.

(A) Barber not Mark. Barber shaves Mark.
(B) Mark shaves the Barber.
(C) Barber doesn't shave themself.
(D) Barber shaves themself.

Its all true. It's all a problem.

Generalized Continuum hypothesis.

There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

The powerset of a set is the set of all subsets.

Recall: powerset of the naturals is not countable.

Resolution of hypothesis?

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Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: "no cardinality between reals and naturals."
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel ..starved himself out of fear of being poisoned..
Russell .. was fine.....but for ...two schizophrenic children..
Dangerous work?

See Logicomix by Doxiaidis, Papadimitriou (was professor here),
Papadatos, Di Donna.

Kolmogorov Complexity, Google, and CS70

Of strings, *s*.

   Minimum sized program that prints string *s*.

What Kolmogorov complexity of a string of 1,000,000, one's?

What is Kolmogorov complexity of a string of *n* one's?

for $i = 1$ to *n*: print '1'.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US?
Acceleration due to gravity on earth?
 Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?
 Kolmorogorov Complexity View:
  perimeter of a circle/diameter.

What is e?
 Kolmorogorov Complexity View(s):
  Continuous Interest Rate:
   $(1 + r/n)^n \to e^r$.
  Solution to: $dy/dx = y$,
   $y \approx (1 + \frac{1}{n})^n)^x \to e^x$.
   Population growth. Covid.

Calculus: what is minimum you need to know?
 Depends on your skills!
 Conceptualization.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.
  Intuition: composition of two linear functions?
    $f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\, x$. Slope is ab.
  Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

  But...but...

For function slopes of tangent differ at different places.

So, where? $f(g(x))$
  slope of f at $g(x)$ times slope of g at x.
  $(f(g(x))' = f'(g(x))g'(x)$.

Product Rule.

Idea: use rise in function value!

$$d(uv) = (u + du)(v + dv) - uv = u\,dv + v\,du + du\,dv \rightarrow u\,dv + v\,du.$$

Any concept:

A quick argument from basic concept of slope of a tangent line.

Perhaps.
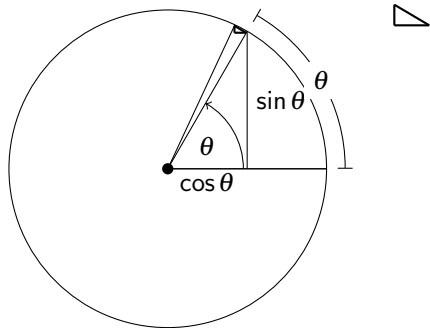
Derivative of sine?

sin(*x*).

What is *x*? An angle in radians.

Let's call it θ and do derivative of $\sin\theta$.

θ - Length of arc of unit circle

Rise. Similar triangle!!! Rise proportional to cosine!

Fundamental Theorem of Calculus.

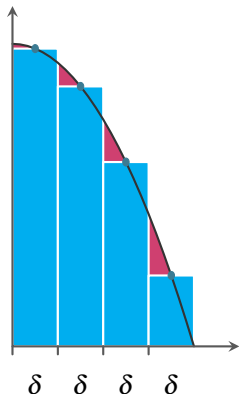Conceptual: Height times Width = Area.

Useful?
  Speed times Time is Distance.

Conceptual: Area is proportional to height.

If you change width, change in area is proportional to height.

Derivative (rate of change) of Area (Integral) under curve, is height of curve.

Calculus

Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$
  "Area is defined as rectangles and add up some thin ones."

Derivative (Rate of change):
  $F'(x) = \lim_{h \to 0} \frac{F(x+h)-F(x)}{h}$.
    "Rise over run of close together points."

Fundamental Theorem: $F(b) - F(a) = \int_a^b F'(x)dx$.
  "Area ($F(\cdot)$) under $f(x)$ grows at x, $F'(x)$, by $f(x)$"
    Thus $F'(x) = f(x)$.

$\delta \quad \delta \quad \delta \quad \delta$

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …...

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Probability: division.

...plus reasoning.

CS 70 : ideas.

Induction $\equiv$ every integer has a next one. Graph theory.
Number of edges is sum of degrees.
$\Delta + 1$ coloring. Neighbors only take up Δ.
Connectivity plus connected components.
Eulerian paths: if you enter you can leave.
Euler's formula: tree has $v - 1$ edges and 1 face plus
each extra edge makes additional face.
$$v - 1 + (f - 1) = e$$

CS 70 : ideas.

Number theory.
  A divisor of x and y divides $x - y$.
  The remainder is always smaller than the divisor.
  $\implies$ Euclid's GCD algorithm.
  Multiplicative Inverse.
  Fermat's theorem from function with inverse is a bijection.
  Gives RSA.

Error Correction.
  (Any) Two points determine a line.
      (well, and d points determine a degree $d + 1$-polynomials.
       Cuz, factoring.
    Find line by linear equations.
   If a couple are wrong, then multiply them by zero, i.e., Error polynomial.

What's going on?

Define. Understand properties. And build from there.

Tools: reasoning, proofs, care.

Gives power to your creativity and in your pursuits.

....and you will pursue probability in this course.