Rao is freaked out.

Are real numbers even real?

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can’t be described.

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

The ratio of the perimeter of a circle to its diameter.

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

The ratio of the perimeter of a circle to its diameter.

e?

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.

$\lim_{n\to\infty}(1+1/n)^n$.

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.

$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$?

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?
The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.
$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.
A solution of $x^2 = 2$.

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?
The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.
$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.
A solution of $x^2 = 2$.

Really, rationals seem fine for...

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?
The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.
$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.
A solution of $x^2 = 2$.

Really, rationals seem fine for... say...

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?
The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.
$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.
A solution of $x^2 = 2$.

Really, rationals seem fine for... say... calculus.

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.

$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.

A solution of $x^2 = 2$.

Really, rationals seem fine for... say... calculus.

$\lim_{n\to\infty}\sum_{i=0}^{n}\frac{(b-a)}{n}f(x_i)$, where $x_i = a + i \times (b-a)/n$.

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.

$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.

A solution of $x^2 = 2$.

Really, rationals seem fine for... say... calculus.

$\lim_{n\to\infty}\sum_{i=0}^{n}\frac{(b-a)}{n}f(x_i)$, where $x_i = a + i \times (b-a)/n$.

So why real numbers?

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?

The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.

$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.

A solution of $x^2 = 2$.

Really, rationals seem fine for... say... calculus.

$\lim_{n\to\infty}\sum_{i=0}^{n}\frac{(b-a)}{n}f(x_i)$, where $x_i = a + i \times (b-a)/n$.

So why real numbers?

$\int_a^b f(x)dx$ is beautiful, succint notation

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?
The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.
$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.
A solution of $x^2 = 2$.

Really, rationals seem fine for... say... calculus.

$\lim_{n\to\infty}\sum_{i=0}^{n}\frac{(b-a)}{n}f(x_i)$, where $x_i = a + i \times (b-a)/n$.

So why real numbers?

$\int_a^b f(x)dx$ is beautiful, succint notation
for a beautiful, succint, powerful idea.

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?
The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.
$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.
A solution of $x^2 = 2$.

Really, rationals seem fine for... say... calculus.

$\lim_{n\to\infty}\sum_{i=0}^{n}\frac{(b-a)}{n}f(x_i)$, where $x_i = a + i \times (b-a)/n$.

So why real numbers?

$\int_a^b f(x)dx$ is beautiful, succint notation
for a beautiful, succint, powerful idea.

What's the idea?

Rao is freaked out.

Are real numbers even real?

Almost all real numbers can't be described.

π?
The ratio of the perimeter of a circle to its diameter.

e? Transcendental number.
$\lim_{n\to\infty}(1+1/n)^n$.

$\sqrt{2}$? Algebraic number.
A solution of $x^2 = 2$.

Really, rationals seem fine for... say... calculus.

$\lim_{n\to\infty}\sum_{i=0}^{n}\frac{(b-a)}{n}f(x_i)$, where $x_i = a + i \times (b-a)/n$.

So why real numbers?

$\int_a^b f(x)dx$ is beautiful, succint notation
for a beautiful, succint, powerful idea.

What's the idea? Area.

Generalized Continuum hypothesis.

There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

Generalized Continuum hypothesis.

There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

The powerset of a set is the set of all subsets.

Resolution of hypothesis?

Resolution of hypothesis?

Gödel. 1940.
Can’t use math!

Resolution of hypothesis?

Gödel. 1940.
Can’t use math!
If math doesn’t contain a contradiction.

Resolution of hypothesis?

Gödel. 1940.
Can’t use math!
If math doesn’t contain a contradiction.

This statement is a lie.

Resolution of hypothesis?

Gödel. 1940.
Can’t use math!
If math doesn’t contain a contradiction.

This statement is a lie.

Is the statement above true?

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Can a program refer to a program?

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Can a program refer to a program?

Can a program refer to itself?

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Can a program refer to a program?

Can a program refer to itself?

Uh oh....

The Barber!

The barber shaves every person who does not shave themselves.

The Barber!

The barber shaves every person who does not shave themselves.

(A) Barber not Mark. Barber shaves Mark.
(B) Mark shaves the Barber.
(C) Barber doesn't shave themself.
(D) Barber shaves themself.

The Barber!

The barber shaves every person who does not shave themselves.

(A) Barber not Mark. Barber shaves Mark.
(B) Mark shaves the Barber.
(C) Barber doesn't shave themself.
(D) Barber shaves themself.

Its all true.

The Barber!

The barber shaves every person who does not shave themselves.

(A) Barber not Mark. Barber shaves Mark.
(B) Mark shaves the Barber.
(C) Barber doesn't shave themself.
(D) Barber shaves themself.

Its all true. It's all a problem.

Generalized Continuum hypothesis.

There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

Generalized Continuum hypothesis.

There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

The powerset of a set is the set of all subsets.

Generalized Continuum hypothesis.

There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

The powerset of a set is the set of all subsets.

Recall: powerset of the naturals is not countable.

Resolution of hypothesis?

Resolution of hypothesis?

Gödel. 1940.
Can’t use math!

Resolution of hypothesis?

Gödel. 1940.
Can’t use math!
If math doesn’t contain a contradiction.

Resolution of hypothesis?

Gödel. 1940.
Can’t use math!
If math doesn’t contain a contradiction.

This statement is a lie.

Resolution of hypothesis?

Gödel. 1940.
Can’t use math!
If math doesn’t contain a contradiction.

This statement is a lie.

Is the statement above true?

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Can a program refer to a program?

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Can a program refer to a program?

Can a program refer to itself?

Resolution of hypothesis?

Gödel. 1940.
Can't use math!
If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Can a program refer to a program?

Can a program refer to itself?

Uh oh....

Changing Axioms?

Goedel:
Any set of axioms is either

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel ..starved himself out of fear of being poisoned..

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel ..starved himself out of fear of being poisoned..
Russell

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel ..starved himself out of fear of being poisoned..
Russell .. was fine...

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel ..starved himself out of fear of being poisoned..
Russell .. was fine.....but for ...

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel ..starved himself out of fear of being poisoned..
Russell .. was fine.....but for ...two schizophrenic children..

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel ..starved himself out of fear of being poisoned..
Russell .. was fine.....but for ...two schizophrenic children..
Dangerous work?

Changing Axioms?

Goedel:
Any set of axioms is either
inconsistent (can prove false statements) or
incomplete (true statements cannot be proven.)

Concrete example:
Continuum hypothesis: “no cardinatity between reals and naturals.”
Continuum hypothesis not disprovable in ZFC
(Goedel 1940.)

Continuum hypothesis not provable.
(Cohen 1963: only Fields medal in logic)

BTW:
Cantor ..bipolar disorder..
Goedel ..starved himself out of fear of being poisoned..
Russell .. was fine.....but for ...two schizophrenic children..
Dangerous work?

See Logicomix by Doxiaidis, Papadimitriou (was professor here),
Papadatos, Di Donna.

Kolmogorov Complexity, Google, and CS70

Of strings, s.

Kolmogorov Complexity, Google, and CS70

Of strings, s.

Minimum sized program that prints string s.

Kolmogorov Complexity, Google, and CS70

Of strings, s.

Minimum sized program that prints string s.

What Kolmogorov complexity of a string of 1,000,000, one's?

Kolmogorov Complexity, Google, and CS70

Of strings, s.

Minimum sized program that prints string s.

What Kolmogorov complexity of a string of 1,000,000, one's?

What is Kolmogorov complexity of a string of n one's?

Kolmogorov Complexity, Google, and CS70

Of strings, s.

Minimum sized program that prints string s.

What Kolmogorov complexity of a string of 1,000,000, one's?

What is Kolmogorov complexity of a string of n one's?

for $i = 1$ to n: print '1'.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth?

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun?

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US?

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas?

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.

Plus “how to program”

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.

Plus “how to program” and remembering a bit.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus “how to program” and remembering a bit.

What is π?

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus “how to program” and remembering a bit.

What is π?

Kolmorogorov Complexity View:

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus “how to program” and remembering a bit.

What is π?

Kolmorogorov Complexity View:
perimeter of a circle/diameter.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?

Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?
Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?
Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?
Kolmorogorov Complexity View(s):
Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.
Solution to: $dy/dx = y$,

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?

Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?

Kolmorogorov Complexity View(s):

Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.

Solution to: $dy/dx = y$,
$y \approx (1 + \frac{1}{n})^n)^x \to e^x$.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?
Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?
Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?
Kolmorogorov Complexity View(s):
Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.
Solution to: $dy/dx = y$,
$y \approx (1 + \frac{1}{n})^n)^x \to e^x$.
Population growth.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?

Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?

Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?

Kolmorogorov Complexity View(s):

Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.

Solution to: $dy/dx = y$,
$y \approx (1 + \frac{1}{n})^n)^x \to e^x$.

Population growth. Covid.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?
Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?
Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?
Kolmorogorov Complexity View(s):
Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.
Solution to: $dy/dx = y$,
$y \approx (1 + \frac{1}{n})^n)^x \to e^x$.
Population growth. Covid.

Calculus:

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?
Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?
Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?
Kolmorogorov Complexity View(s):
Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.
Solution to: $dy/dx = y$,
$y \approx (1 + \frac{1}{n})^n)^x \to e^x$.
Population growth. Covid.

Calculus: what is minimum you need to know?

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?
Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?
Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?
Kolmorogorov Complexity View(s):
Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.
Solution to: $dy/dx = y$,
$y \approx (1 + \frac{1}{n})^n)^x \to e^x$.
Population growth. Covid.

Calculus: what is minimum you need to know?
Depends on your skills!

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?
Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?
Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?
Kolmorogorov Complexity View(s):
Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.
Solution to: $dy/dx = y$,
$y \approx (1 + \frac{1}{n})^n)^x \to e^x$.
Population growth. Covid.

Calculus: what is minimum you need to know?
Depends on your skills!
Conceptualization.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun? Population of the US? Acceleration due to gravity on earth?
Google. Plus reference.

Syntax of pandas? Google + Stackoverflow.
Plus "how to program" and remembering a bit.

What is π?
Kolmorogorov Complexity View:
perimeter of a circle/diameter.

What is e?
Kolmorogorov Complexity View(s):
Continuous Interest Rate:
$(1 + r/n)^n \to e^r$.
Solution to: $dy/dx = y$,
$y \approx (1 + \frac{1}{n})^n)^x \to e^x$.
Population growth. Covid.

Calculus: what is minimum you need to know?
Depends on your skills!
Conceptualization.

Calculus

What is the first half of calculus about?

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule?

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.
Intuition: composition of two linear functions?

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

For function slopes of tangent differ at different places.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

For function slopes of tangent differ at different places.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

For function slopes of tangent differ at different places.

So, where?

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

For function slopes of tangent differ at different places.

So, where? $f(g(x))$

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

For function slopes of tangent differ at different places.

So, where? $f(g(x))$

slope of f at $g(x)$ times slope of g at x.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

For function slopes of tangent differ at different places.

So, where? $f(g(x))$

slope of f at $g(x)$ times slope of g at x.

$(f(g(x))' = f'(g(x))g'(x)$.

Calculus

What is the first half of calculus about?

The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

For function slopes of tangent differ at different places.

So, where? $f(g(x))$

slope of f at $g(x)$ times slope of g at x.

$(f(g(x))' = f'(g(x))g'(x)$.

Product Rule.

Idea: use rise in function value!

Product Rule.

Idea: use rise in function value!

$d(uv) = (u+du)(v+dv) - uv = udv + vdu + dudv \to udv + vdu$.

Product Rule.

Idea: use rise in function value!

$d(uv) = (u + du)(v + dv) - uv = udv + vdu + dudv \to udv + vdu$.

Any concept:

Product Rule.

Idea: use rise in function value!

$d(uv) = (u + du)(v + dv) - uv = udv + vdu + dudv \to udv + vdu$.

Any concept:

A quick argument from basic concept of slope of a tangent line.

Product Rule.

Idea: use rise in function value!

$d(uv) = (u + du)(v + dv) - uv = udv + vdu + dudv \to udv + vdu$.

Any concept:

A quick argument from basic concept of slope of a tangent line.

Perhaps.

Derivative of sine?

$\sin(x)$.

Derivative of sine?

$\sin(x)$.

What is x? An angle in radians.

Derivative of sine?

$\sin(x)$.

What is x? An angle in radians.

Let's call it θ and do derivative of $\sin\theta$.

Derivative of sine?

$\sin(x)$.

What is x? An angle in radians.

Let's call it θ and do derivative of $\sin\theta$.

θ - Length of arc of unit circle

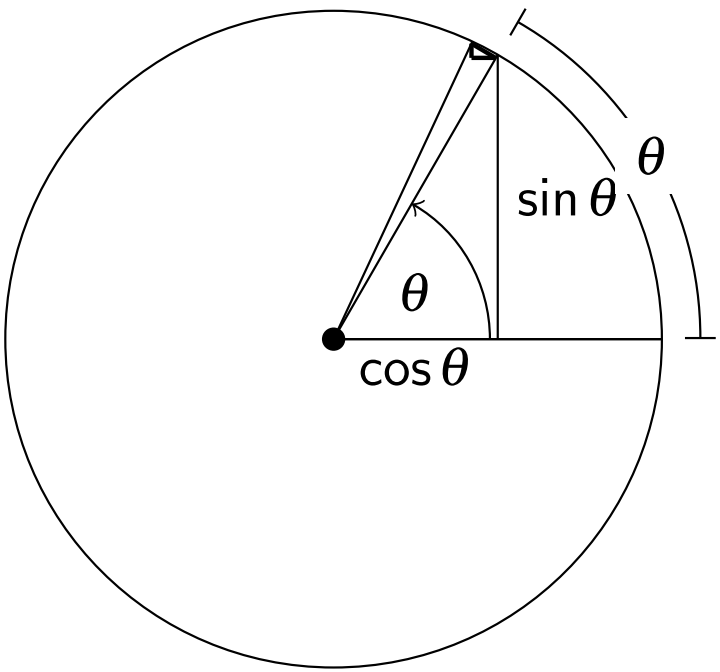
Derivative of sine?

$\sin(x)$.

What is x? An angle in radians.

Let's call it θ and do derivative of $\sin\theta$.

θ - Length of arc of unit circle



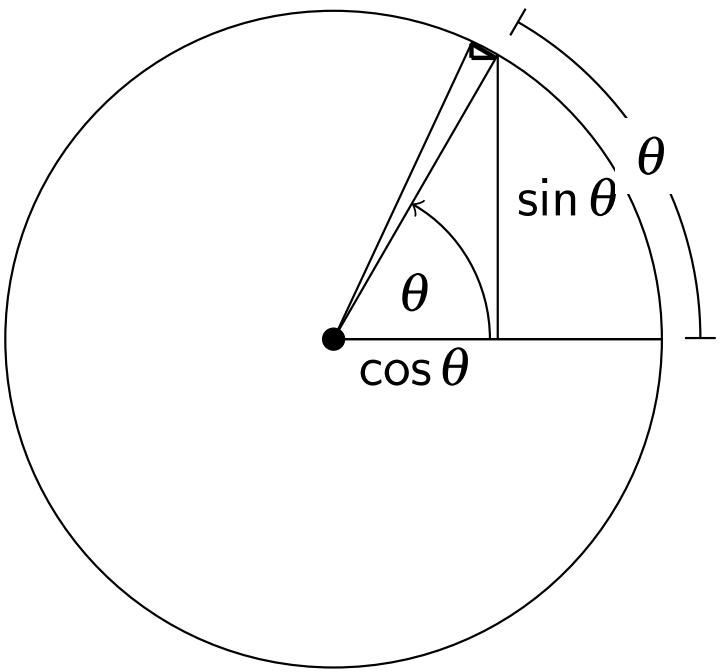
Derivative of sine?

$\sin(x)$.

What is x? An angle in radians.

Let's call it θ and do derivative of $\sin\theta$.

θ - Length of arc of unit circle



Rise.

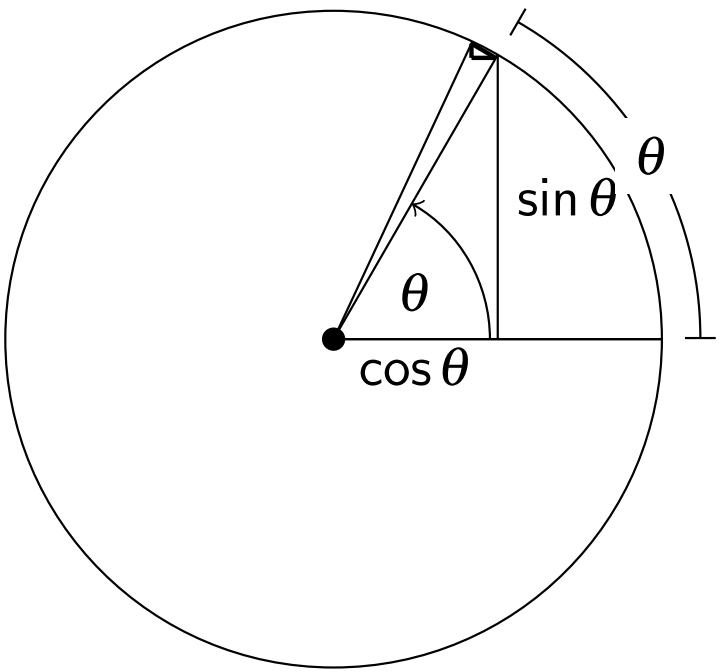
Derivative of sine?

$\sin(x)$.

What is x? An angle in radians.

Let's call it θ and do derivative of $\sin\theta$.

θ - Length of arc of unit circle



Rise. Similar triangle!!!

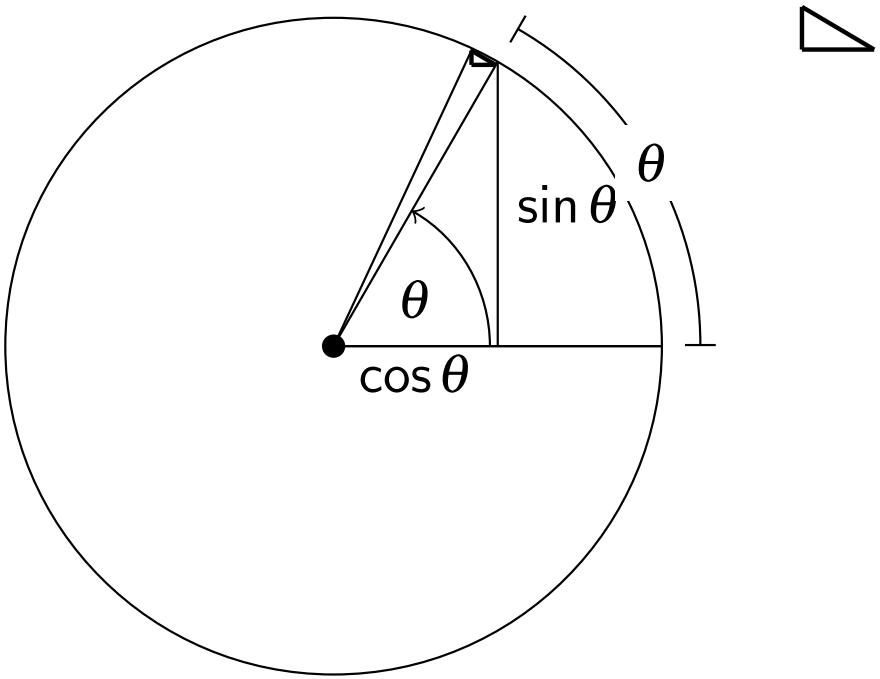
Derivative of sine?

$\sin(x)$.

What is x? An angle in radians.

Let's call it θ and do derivative of $\sin\theta$.

θ - Length of arc of unit circle



Rise. Similar triangle!!! Rise proportional to cosine!

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

Useful?

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

Useful?

Speed times Time is Distance.

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

Useful?
Speed times Time is Distance.

Conceptual: Area is proportional to height.

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

Useful?
Speed times Time is Distance.

Conceptual: Area is proportional to height.

If you change width, change in area is proportional to height.

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

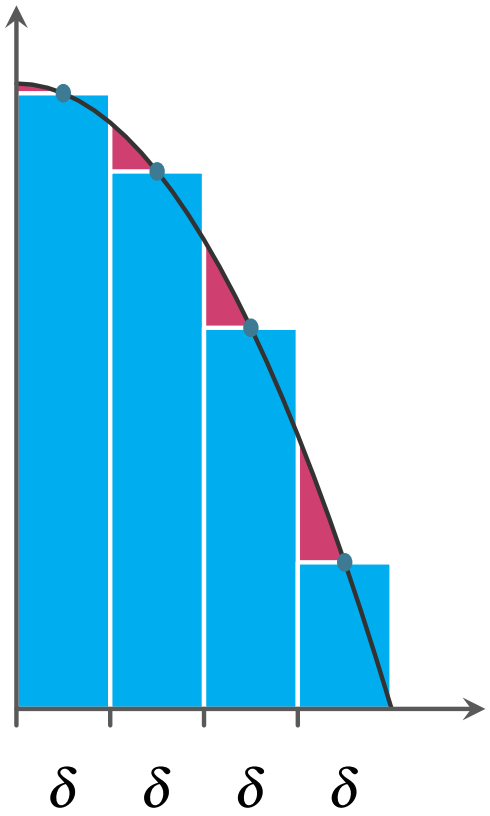
Useful?
Speed times Time is Distance.

Conceptual: Area is proportional to height.

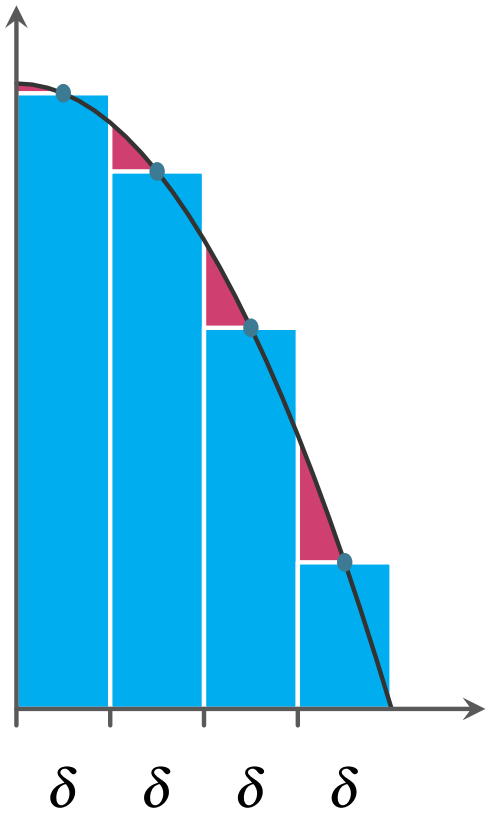
If you change width, change in area is proportional to height.

Derivative (rate of change) of Area (Integral) under curve, is height of curve.

Calculus

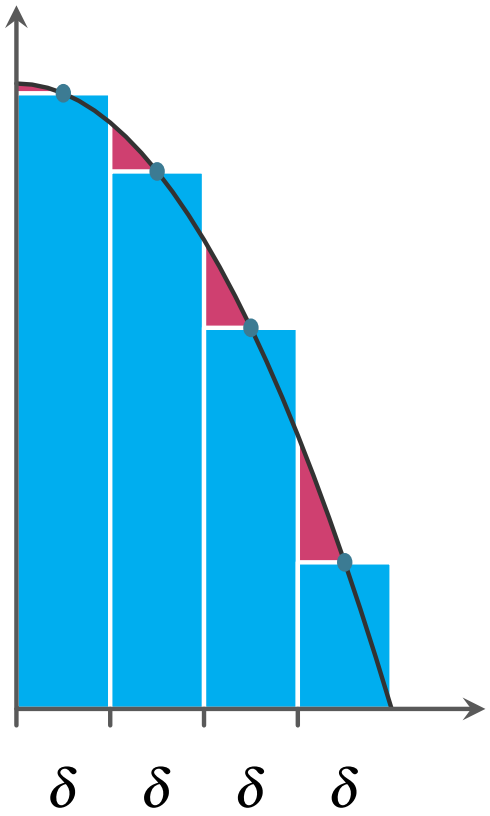


Calculus



Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

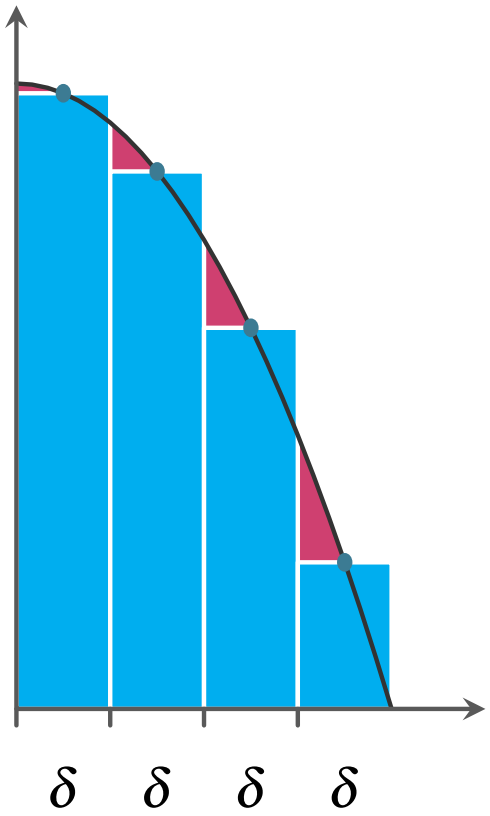
Calculus



Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

"Area is defined as rectangles and add up some thin ones."

Calculus



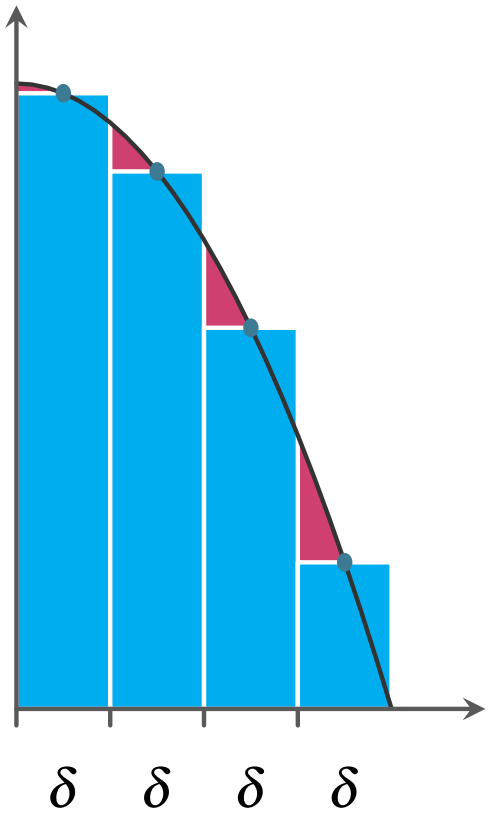
Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

"Area is defined as rectangles and add up some thin ones."

Derivative (Rate of change):

$F'(x) = \lim_{h \to 0} \frac{F(x+h)-F(x)}{h}$.

Calculus



Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

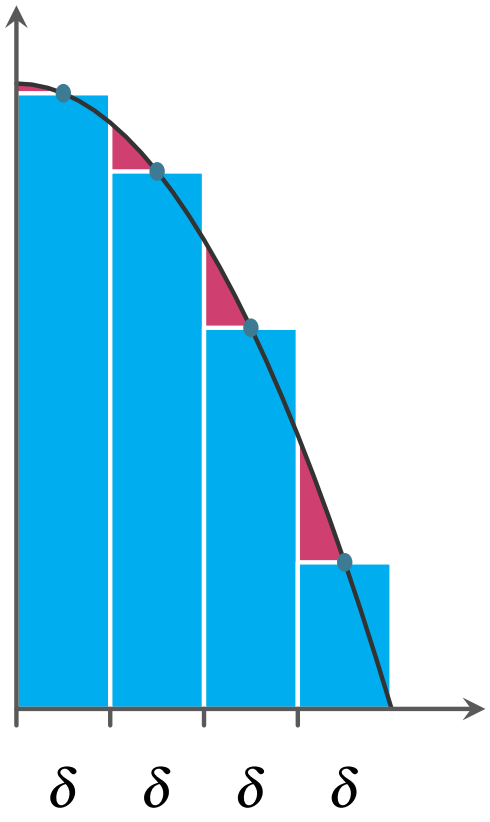
"Area is defined as rectangles and add up some thin ones."

Derivative (Rate of change):

$F'(x) = \lim_{h \to 0} \frac{F(x+h)-F(x)}{h}$.

"Rise over run of close together points."

Calculus



Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

"Area is defined as rectangles and add up some thin ones."

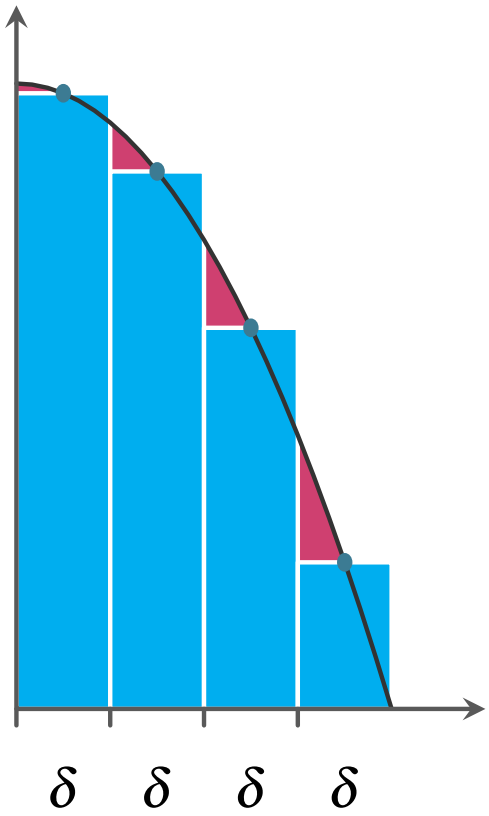
Derivative (Rate of change):

$F'(x) = \lim_{h \to 0} \frac{F(x+h)-F(x)}{h}$.

"Rise over run of close together points."

Fundamental Theorem: $F(b) - F(a) = \int_a^b F'(x)dx$.

Calculus



Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

"Area is defined as rectangles and add up some thin ones."

Derivative (Rate of change):

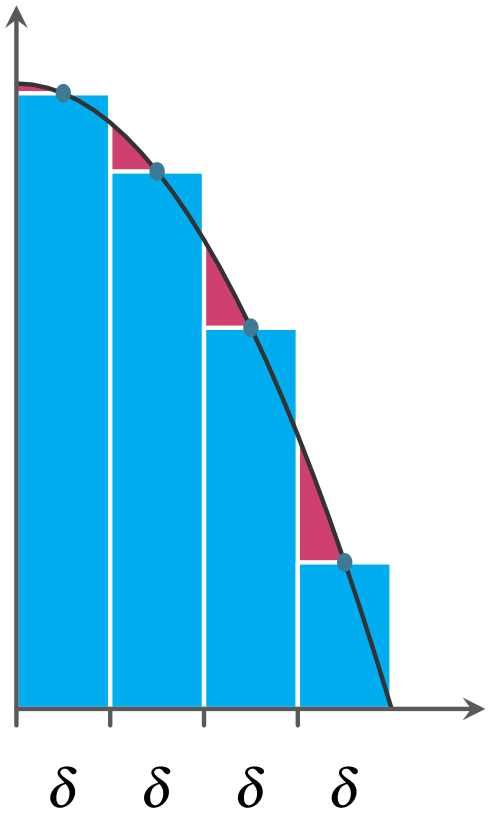
$F'(x) = \lim_{h \to 0} \frac{F(x+h)-F(x)}{h}$.

"Rise over run of close together points."

Fundamental Theorem: $F(b) - F(a) = \int_a^b F'(x)dx$.

"Area ($F(\cdot)$) under $f(x)$ grows at x, $F'(x)$, by $f(x)$"

Calculus



Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

"Area is defined as rectangles and add up some thin ones."

Derivative (Rate of change):

$F'(x) = \lim_{h \to 0} \frac{F(x+h)-F(x)}{h}$.

"Rise over run of close together points."

Fundamental Theorem: $F(b) - F(a) = \int_a^b F'(x)dx$.

"Area ($F(\cdot)$) under $f(x)$ grows at x, $F'(x)$, by $f(x)$"

Thus $F'(x) = f(x)$.

Arguments, reasoning.

What you know: slope, limit.

Arguments, reasoning.

What you know: slope, limit.

Plus: definition.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ….

Knowing how to program plus some syntax (google) gives the ability to program.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ...

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Probability:

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Probability: division.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ...

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Probability: division.

...plus reasoning.

CS 70 : ideas.

Induction

CS 70 : ideas.

Induction $\equiv$ every integer has a next one.

CS 70 : ideas.

Induction $\equiv$ every integer has a next one. Graph theory.
Number of edges is sum of degrees.
$\Delta + 1$ coloring. Neighbors only take up Δ.
Connectivity plus connected components.
Eulerian paths: if you enter you can leave.
Euler's formula: tree has $v - 1$ edges and 1 face plus
each extra edge makes additional face.
$v - 1 + (f - 1) = e$

CS 70 : ideas.

Number theory.

A divisor of x and y divides $x - y$.

The remainder is always smaller than the divisor.

$\Longrightarrow$ Euclid's GCD algorithm.

Multiplicative Inverse.

Fermat's theorem from function with inverse is a bijection.

Gives RSA.

CS 70 : ideas.

Number theory.

A divisor of x and y divides $x - y$.

The remainder is always smaller than the divisor.

$\Longrightarrow$ Euclid's GCD algorithm.

Multiplicative Inverse.

Fermat's theorem from function with inverse is a bijection.

Gives RSA.

Error Correction.

(Any) Two points determine a line.

(well, and d points determine a degree $d+1$-polynomials.

Cuz, factoring.

Find line by linear equations.

If a couple are wrong, then multiply them by zero, i.e., Error polynomial.

CS70 and your future?

What's going on?

CS70 and your future?

What's going on?

Define. Understand properties. And build from there.

CS70 and your future?

What's going on?

Define. Understand properties. And build from there.

Tools: reasoning, proofs, care.

CS70 and your future?

What's going on?

Define. Understand properties. And build from there.

Tools: reasoning, proofs, care.

CS70 and your future?

What's going on?

Define. Understand properties. And build from there.

Tools: reasoning, proofs, care.

Gives power to your creativity and in your pursuits.

CS70 and your future?

What's going on?

Define. Understand properties. And build from there.

Tools: reasoning, proofs, care.

Gives power to your creativity and in your pursuits.

....and you will pursue probability in this course.