Lecture 5: Graphs.

Graphs!

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Graphs!
Definitions: model.

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Graphs!
  Definitions: model.
  Fact!

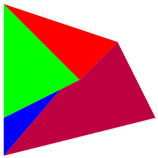
Lecture 5: Graphs.

Graphs!
  Definitions: model.
  Fact!

Lecture 5: Graphs.

Graphs!
- Definitions: model.
- Fact!

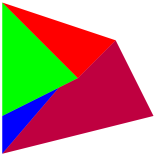
Map Coloring.

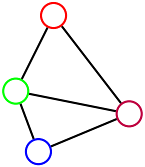


Map Coloring.

Map Coloring.

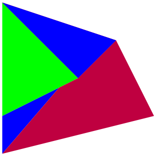
Map Coloring.

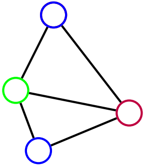




Fewer Colors?

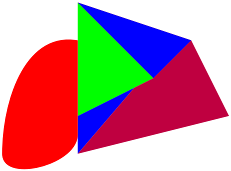
Map Coloring.

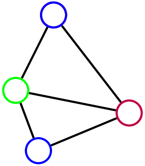




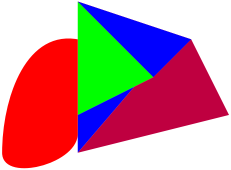
Yes! Three colors.

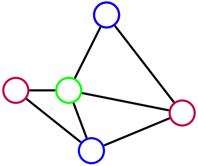
Map Coloring.



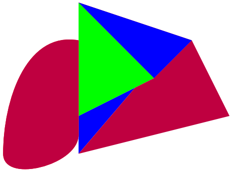


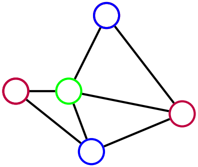
Map Coloring.



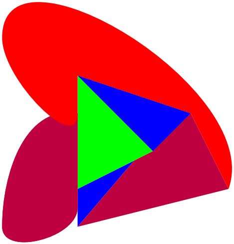


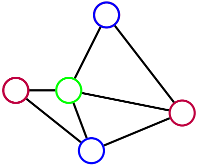
Map Coloring.



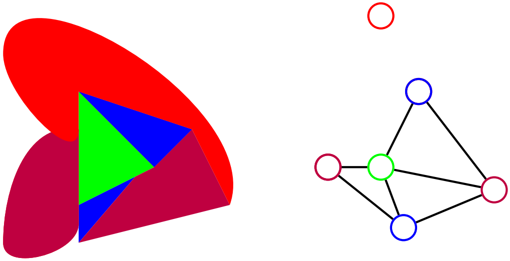


Map Coloring.

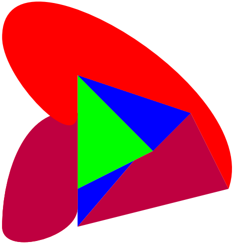


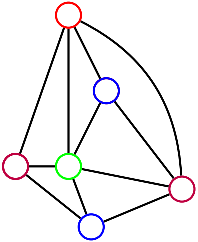


Map Coloring.

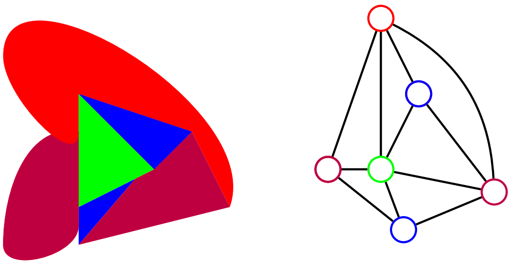


Map Coloring.



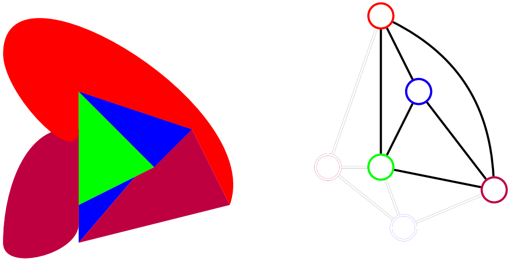


Map Coloring.

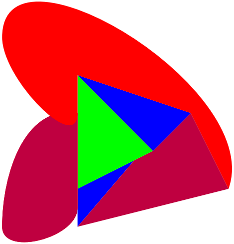


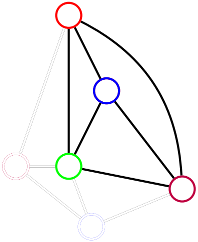
Fewer Colors?

Map Coloring.



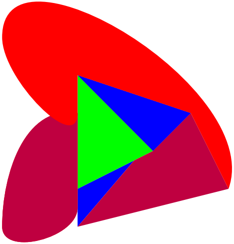
Map Coloring.

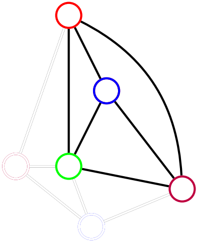




Four colors required!

Map Coloring.

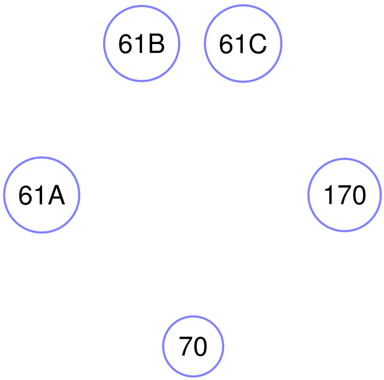




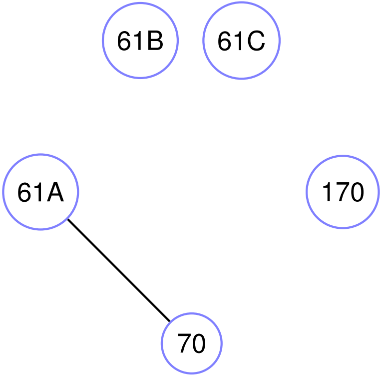
Four colors required!

Theorem: Four colors enough for maps on the plane.

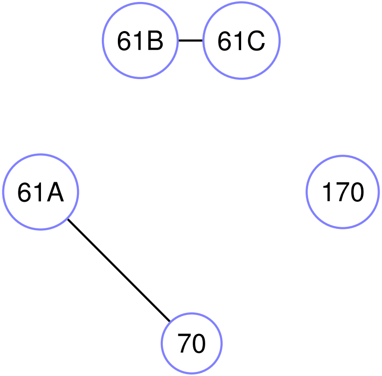
Scheduling: coloring.



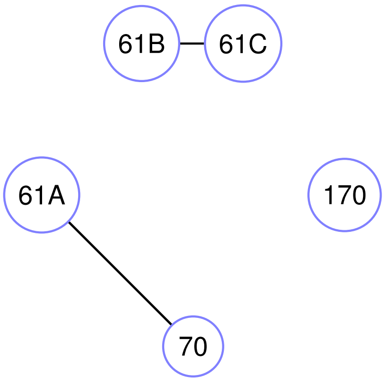
Scheduling: coloring.



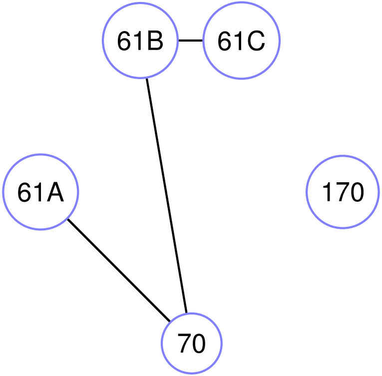
Scheduling: coloring.



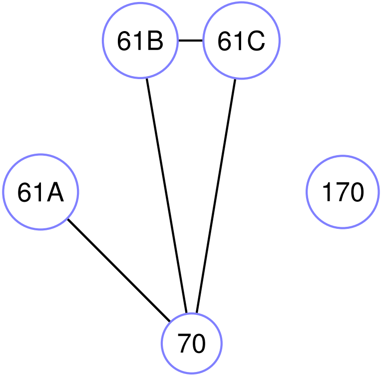
Scheduling: coloring.



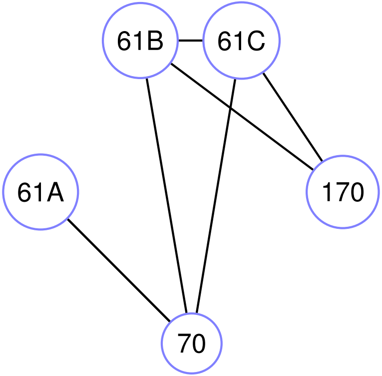
Scheduling: coloring.



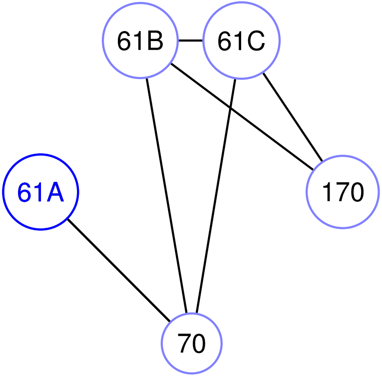
Scheduling: coloring.



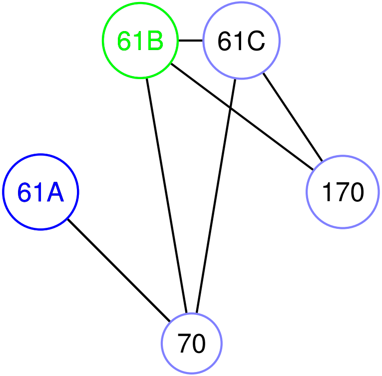
Scheduling: coloring.



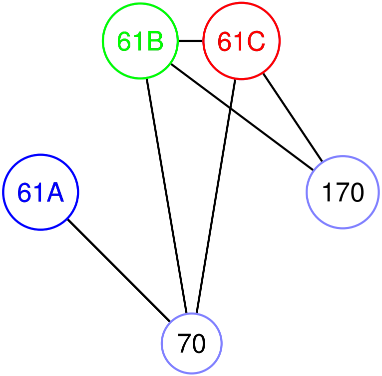
Scheduling: coloring.



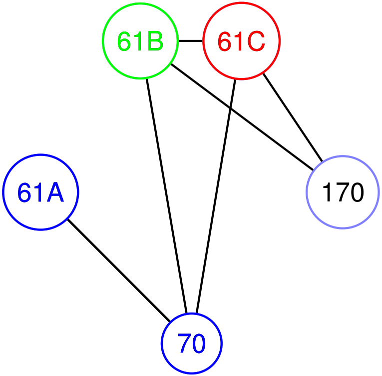
Scheduling: coloring.



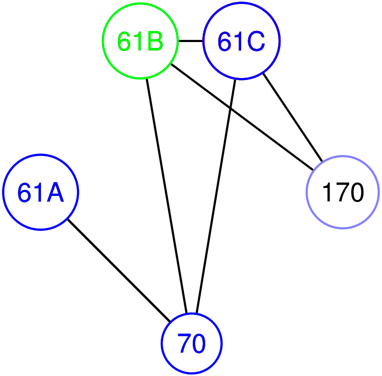
Scheduling: coloring.



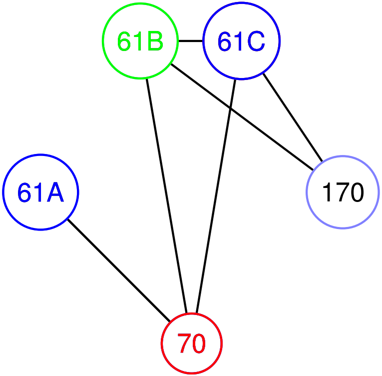
Scheduling: coloring.



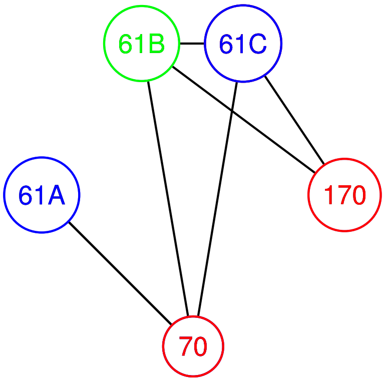
Scheduling: coloring.



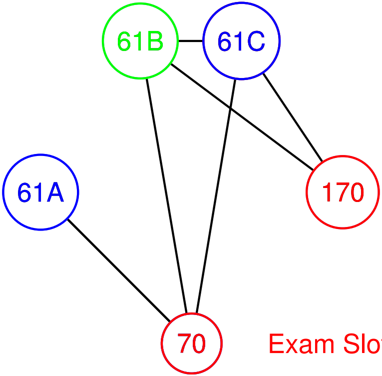
Scheduling: coloring.



Scheduling: coloring.



Scheduling: coloring.

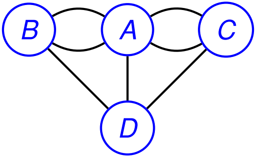


Exam Slot 1.

Exam Slot 2.

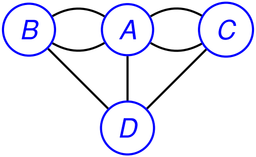
Exam Slot 3.

Graphs: formally.



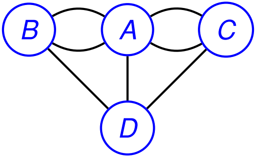
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

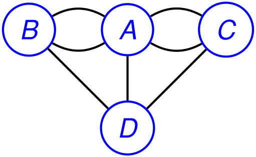
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

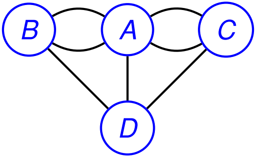


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



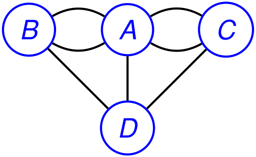
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



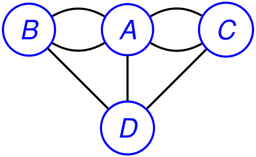
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

Graphs: formally.



Graph: $G = (V, E)$.

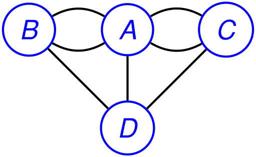
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

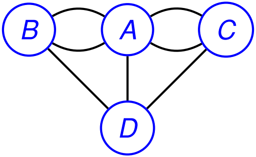
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

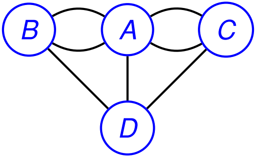
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.



Graph: $G = (V, E)$.

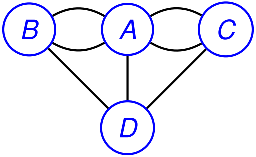
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

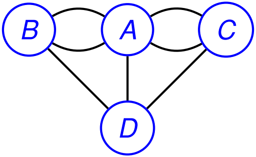
$\{A, B, C, D\}$

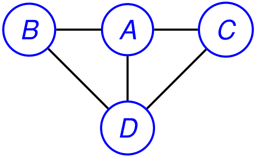
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A,B\},\{A,B\},\{A,C\},\{A,C\},\{B,D\},\{A,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.

Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

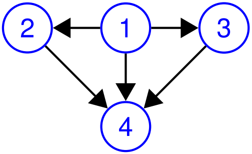
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

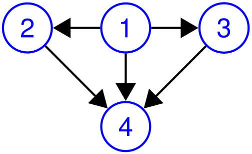
Multigraph above.

Directed Graphs



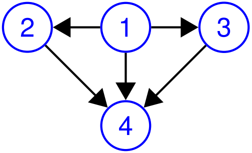
$G = (V, E)$.

Directed Graphs



$G = (V, E)$.
V - set of vertices.

Directed Graphs

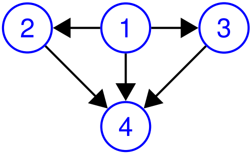


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



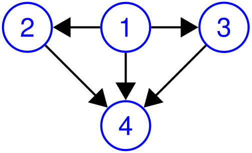
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



$G = (V, E)$.

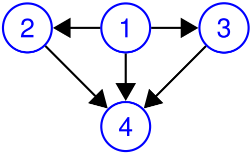
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



$G = (V, E)$.

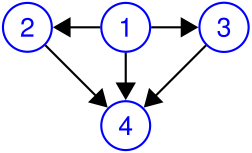
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



$G = (V, E)$.

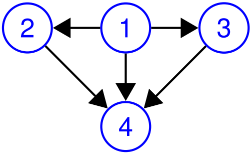
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4),$

Directed Graphs



$G = (V, E)$.

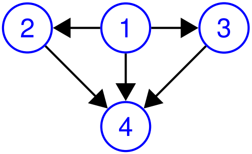
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

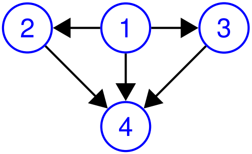
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Directed Graphs



$G = (V, E)$.

V - set of vertices.

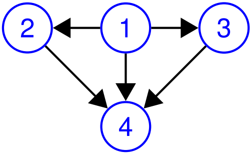
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament:

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

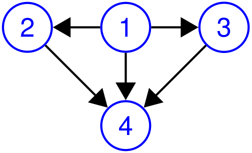
E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.

Tournament: 1 beats 2,

Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
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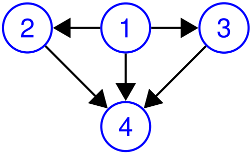
One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



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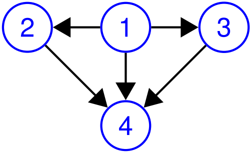

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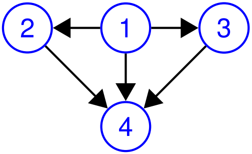

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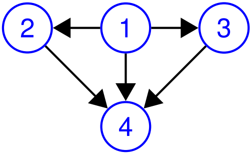
Social Network:

Directed Graphs



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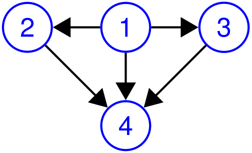
Social Network: Directed?

Directed Graphs



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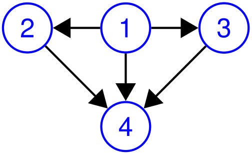
Social Network: Directed? Undirected?

Directed Graphs



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One way streets.
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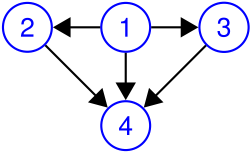
Social Network: Directed? Undirected?
Friends.

Directed Graphs



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Tournament: 1 beats 2, ...
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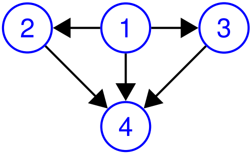
Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs



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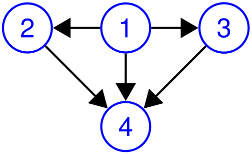
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs



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Tournament: 1 beats 2, ...
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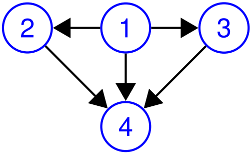
Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



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Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

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neighbors, adjacent, degree, incident, in-degree, out-degree

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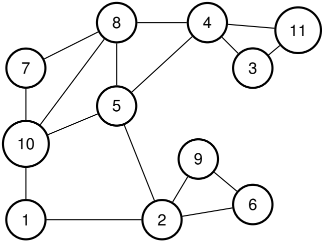
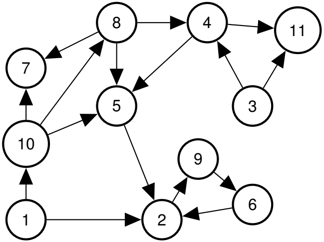


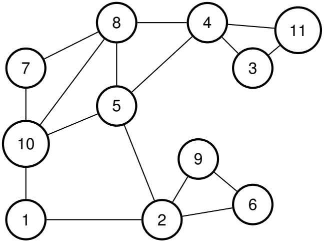


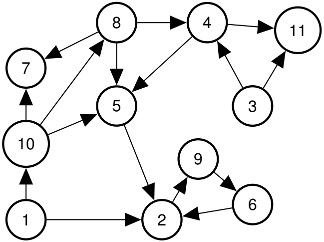
Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



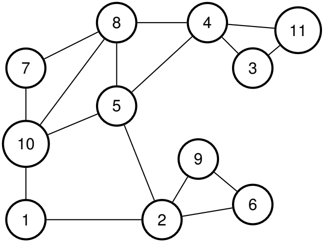


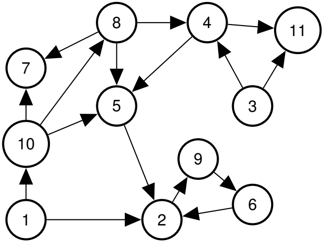
Neighbors of 10? 1,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1,5,

Graph Concepts and Definitions.

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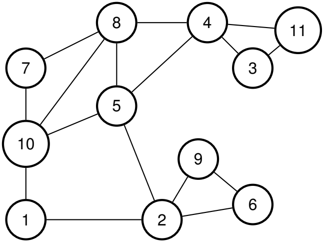

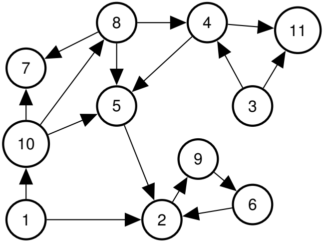

Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

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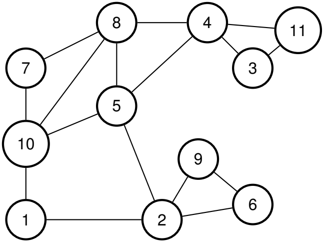

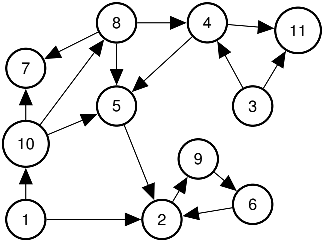

Neighbors of 10? 1,5,7, 8.

Graph Concepts and Definitions.

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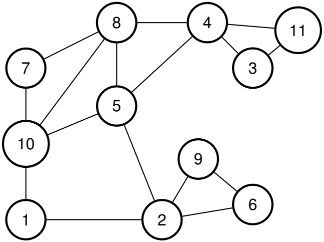
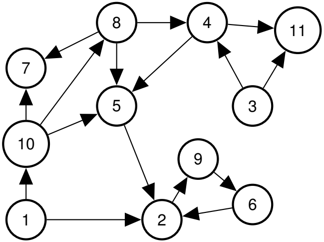
neighbors, adjacent, degree, incident, in-degree, out-degree





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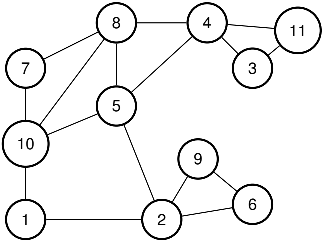

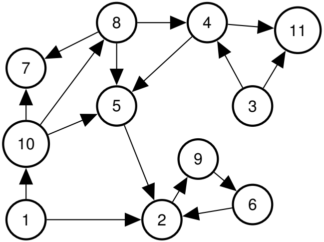

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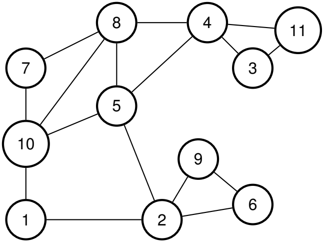

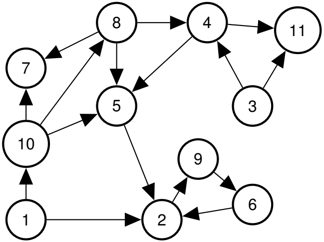

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Degree of vertex 1?

Graph Concepts and Definitions.

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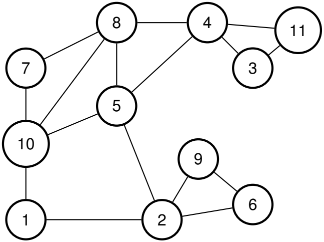

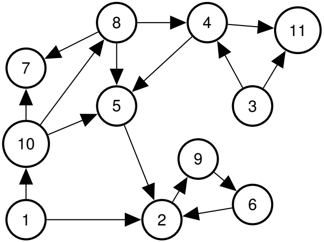

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Degree of vertex 1? 2

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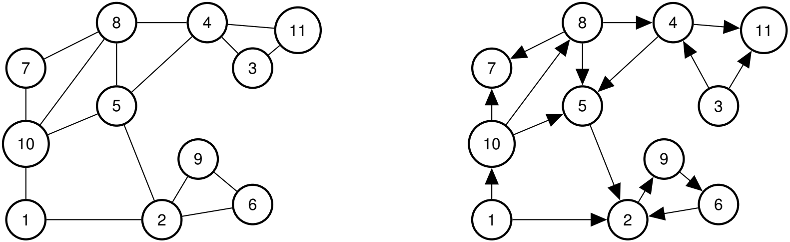
neighbors, adjacent, degree, incident, in-degree, out-degree



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Degree of vertex 1? 2

Degree of vertex u is number of incident edges.

Graph Concepts and Definitions.

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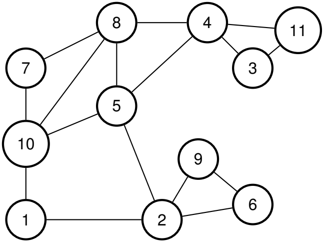

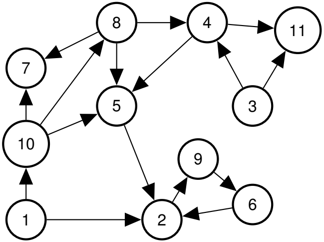

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Equals number of neighbors in simple graph.

Graph Concepts and Definitions.

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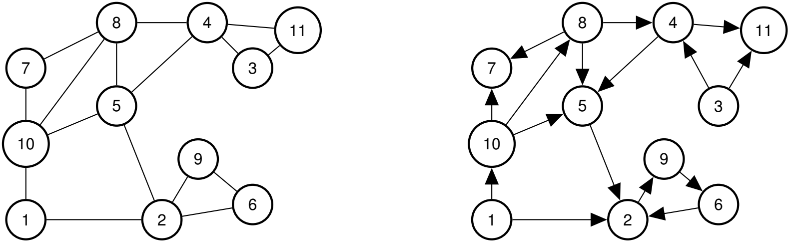
neighbors, adjacent, degree, incident, in-degree, out-degree



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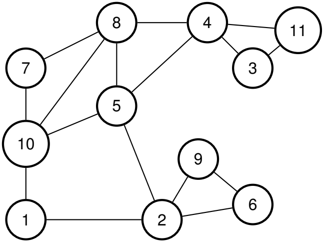

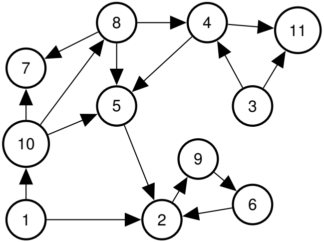

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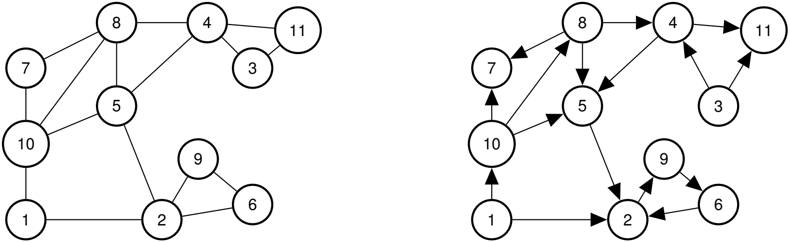
Directed graph?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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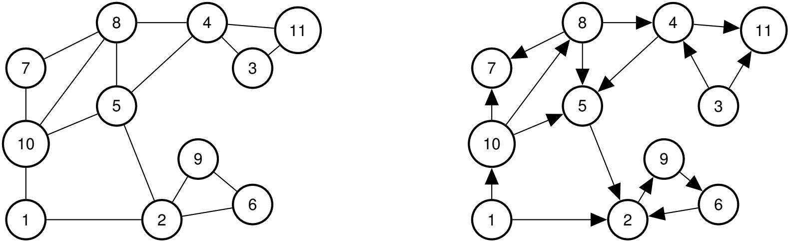
Directed graph?

In-degree of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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Directed graph?

In-degree of 10? 1

Graph Concepts and Definitions.

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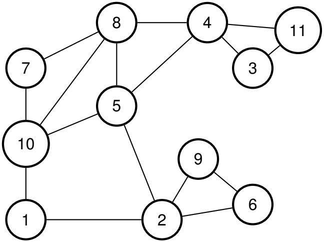

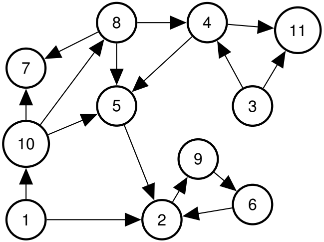

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Directed graph?

In-degree of 10? 1 Out-degree of 10?

Graph Concepts and Definitions.

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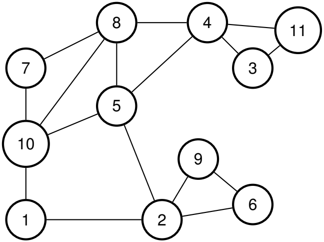

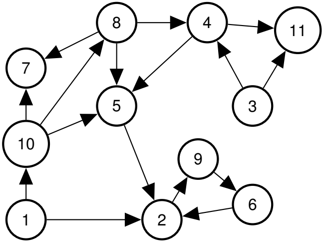

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Directed graph?

In-degree of 10? 1 Out-degree of 10? 3

Graph Concepts and Definitions.

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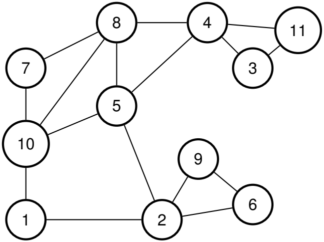

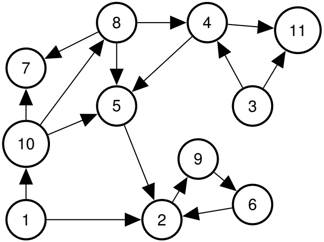

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Directed graph?

In-degree of 10? 1   Out-degree of 10? 3

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

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Graph Concepts and Definitions.

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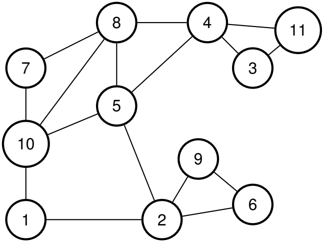
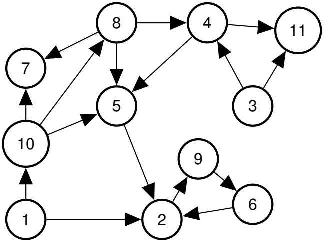




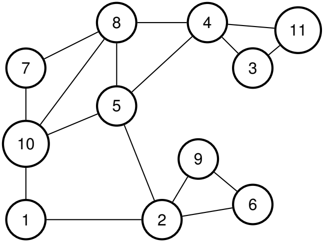
Edge $(8,5)$ **is incident to:**

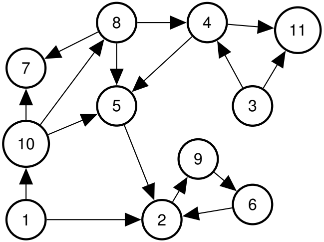
(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8,5)$.
(D) Edge $(8,4)$.
(E) Vertex 10.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Edge $(8, 5)$ **is incident to:**

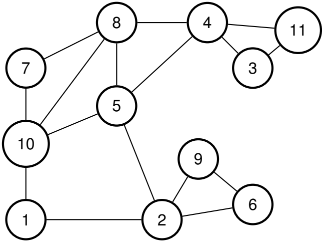
(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8, 5)$.
(D) Edge $(8, 4)$.
(E) Vertex 10.

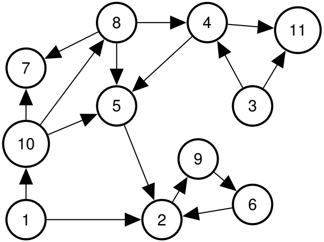
(A) and (B) are true.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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The degree of a vertex is:

(A) The number of edges incident to it.
(B) The number of neighbors of v.
(C) Is the number of vertices in its connected component.

Graph Concepts and Definitions.

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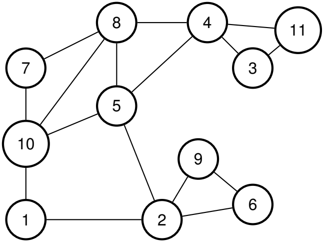

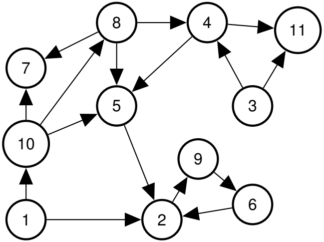

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Sum of degrees?

The sum of the vertex degrees is equal to

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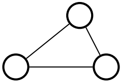
(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)!



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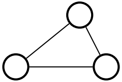
Not (A)! Triangle.



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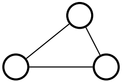

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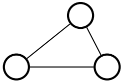

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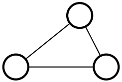

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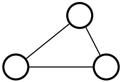

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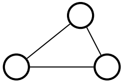

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What? For triangle number of edges is 3, the sum of degrees is 6.

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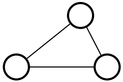

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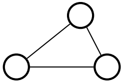

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could sum always be...

(A) $2|E|$? ..

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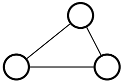

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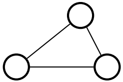

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(A) $2|E|$? ..
(B) $2|V|$?
(A) is true.

Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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The sum of the vertex degrees is equal to ??

Recall:

Quick Proof of an Equality.

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edge, (u, v), is incident to endpoints, u and v.

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degree of u number of edges incident to u

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Let's count incidences in two ways.

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Total Incidences?

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Quick Proof of an Equality.

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Let's count incidences in two ways.

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What is degree v? Incidences corresponding to v!

Quick Proof of an Equality.

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Total Incidences? The sum over vertices of degrees!

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Total Incidences? The sum over vertices of degrees!

Thm: Sum of vertex degress is $2|E|$.

Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

Poll: Proof of “handshake” lemma.

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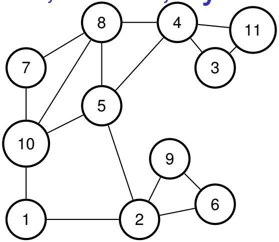
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

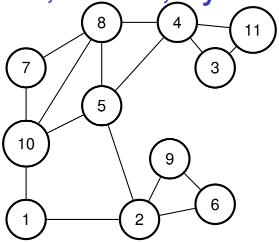
(A),(C), (D), (E), and (F).

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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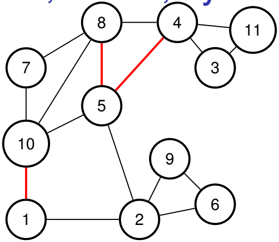
Path?

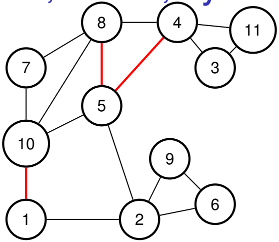
Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$?

Paths, walks, cycles, tour.



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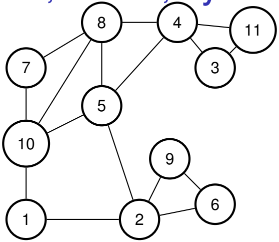

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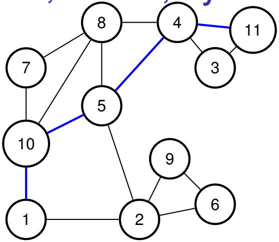
Paths, walks, cycles, tour.

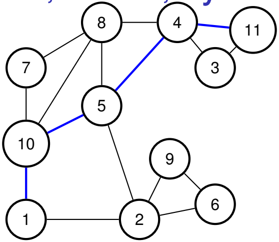


A path in a graph is a sequence of edges.

Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$? No!

Path? $\{1, 10\}$, $\{10, 5\}$, $\{5, 4\}$, $\{4, 11\}$?

Paths, walks, cycles, tour.



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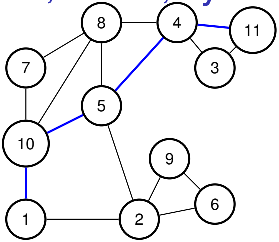

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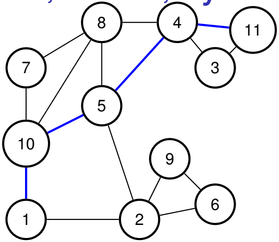
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.

Paths, walks, cycles, tour.



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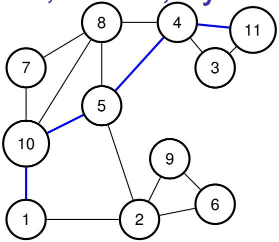
Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1,v_2),(v_2,v_3),\ldots(v_{k-1},v_k)$.

Quick Check!

Paths, walks, cycles, tour.



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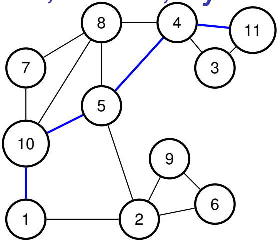
Quick Check! Length of path?

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

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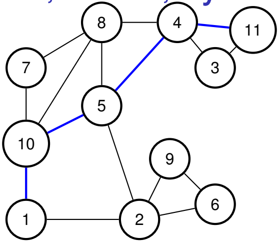
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

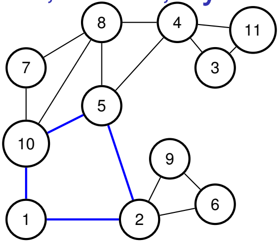
Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1,v_2),(v_2,v_3),\ldots(v_{k-1},v_k)$.

Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

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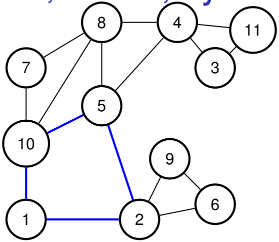
Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.

Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path from v_1 to v_{k-1}, + edge (v_{k-1}, v_1)

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

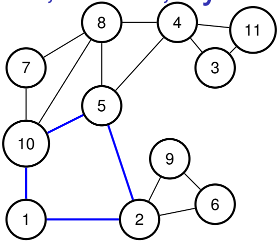
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.

Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path from v_1 to v_{k-1}, + edge (v_{k-1}, v_1) Length of cycle?

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

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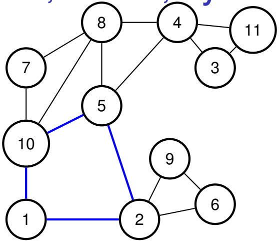

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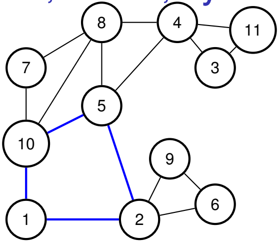

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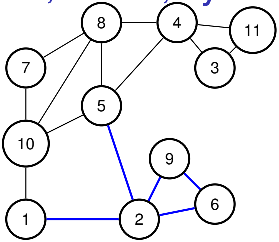

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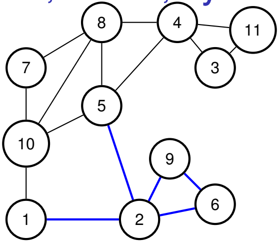

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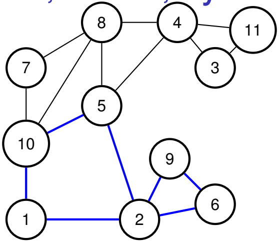

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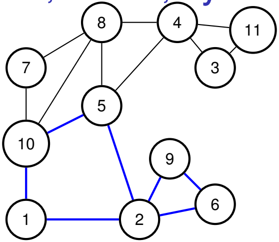

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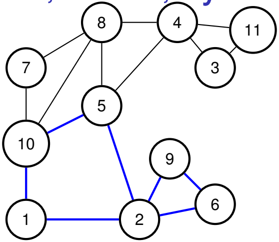

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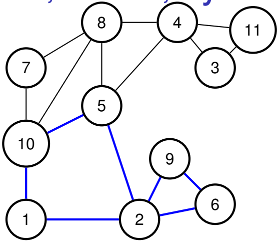

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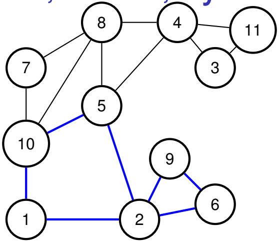

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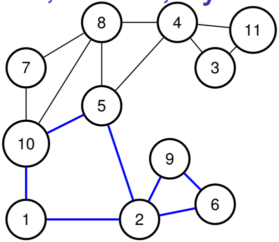

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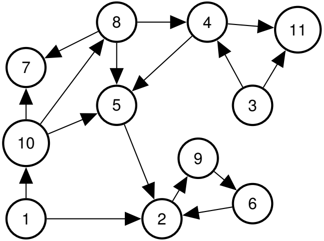

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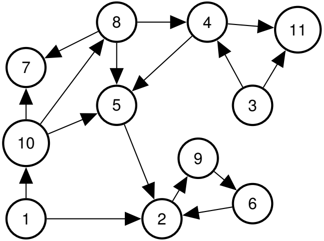

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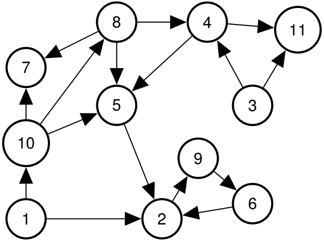

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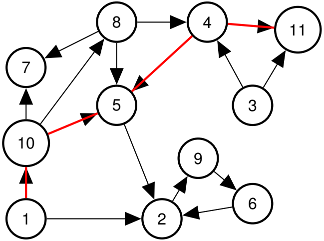

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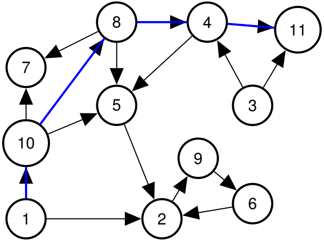

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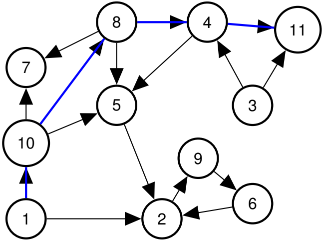

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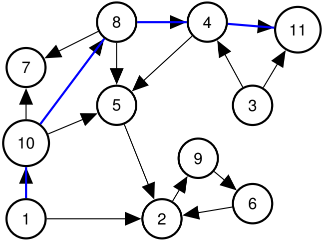

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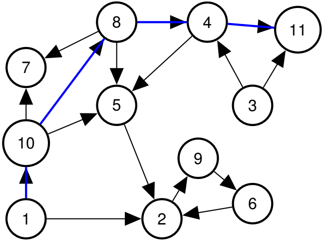

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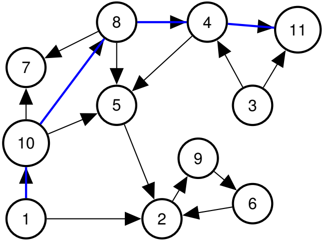

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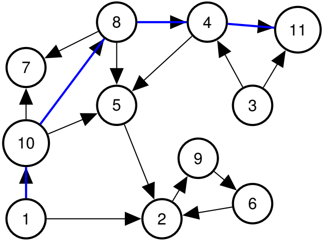

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Paths, walks, cycles, tours ... are analagous to undirected now.

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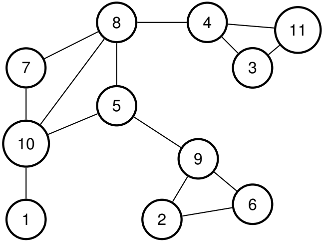

u and v are connected if there is a path between u and v.

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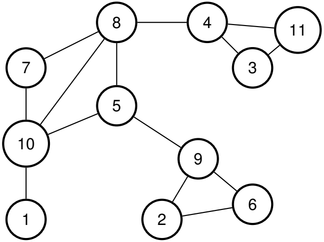

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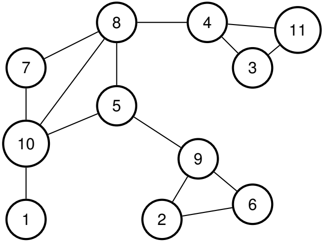

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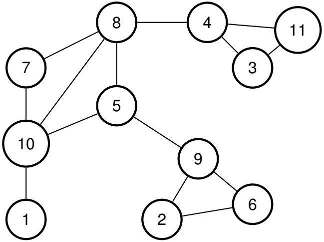

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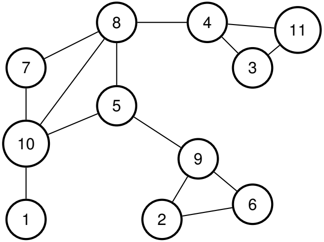

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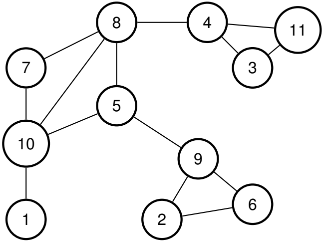

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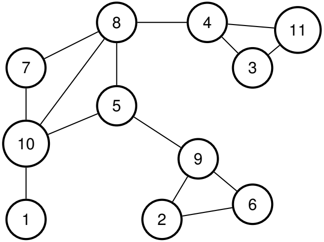

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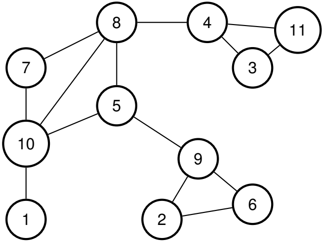

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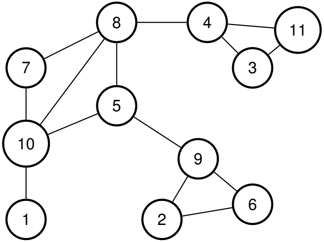

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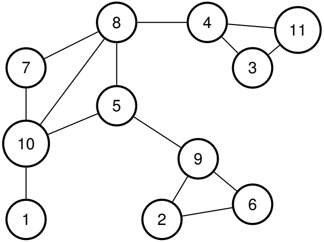

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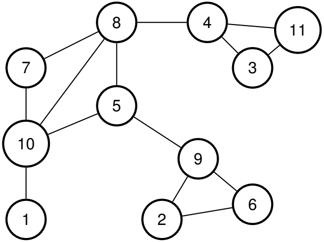

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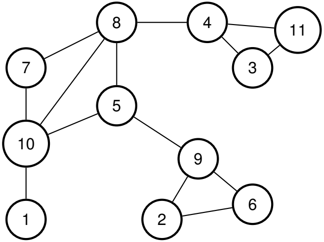

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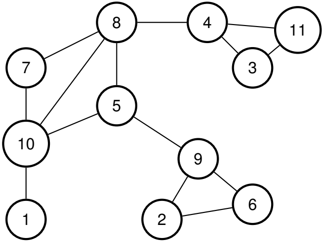

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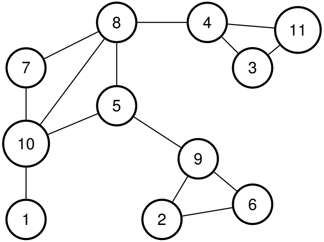

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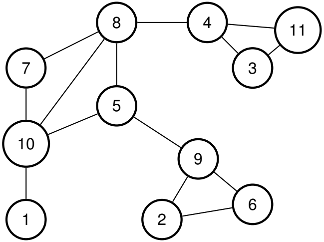

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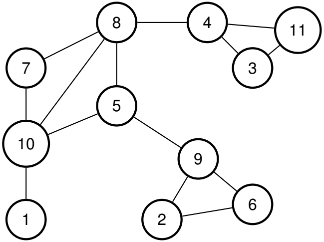

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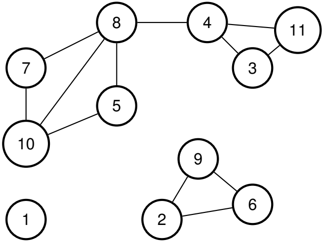

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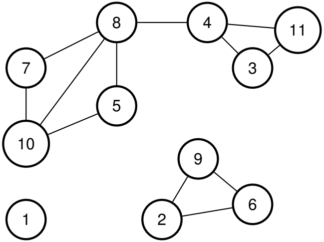

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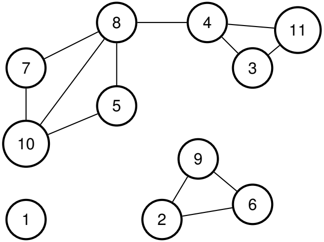

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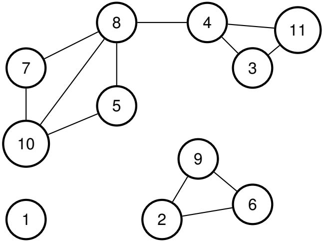

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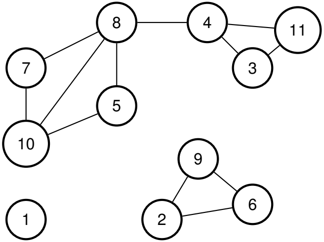
Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected Components: Quiz.



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Connected component - maximal set of connected vertices.

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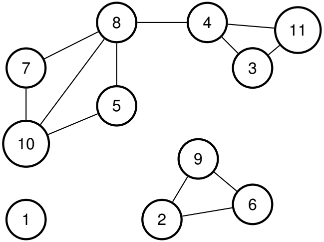

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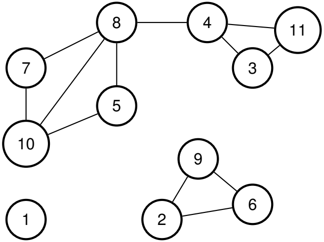
Quick Check: Is $\{10, 7, 5\}$ a connected component?

Connected Components: Quiz.



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Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.
Connected component - maximal set of connected vertices.
Quick Check: Is $\{10, 7, 5\}$ a connected component? No.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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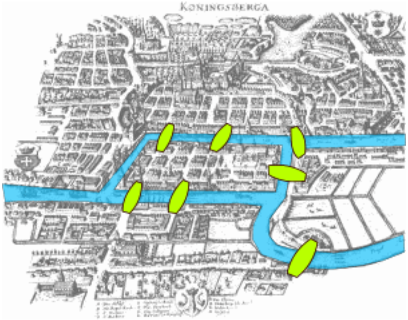

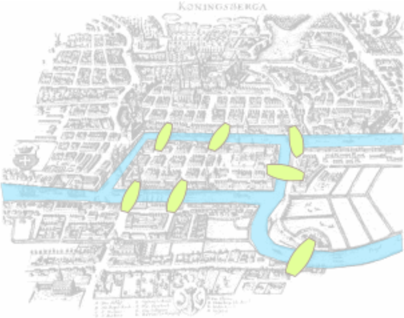

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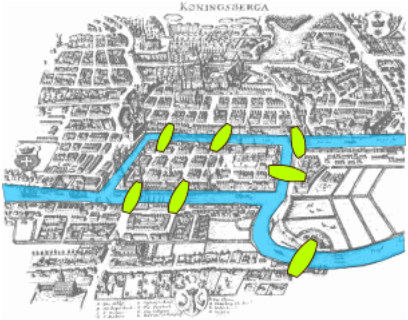

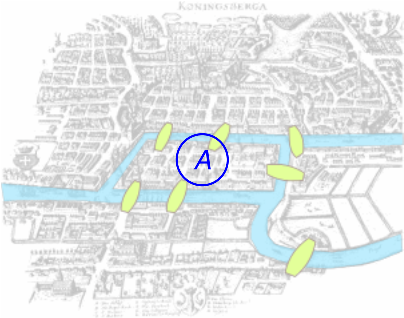

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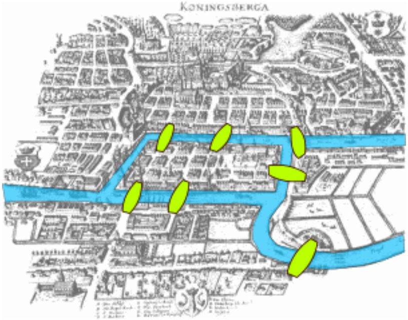

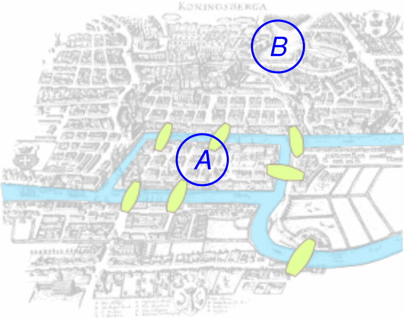

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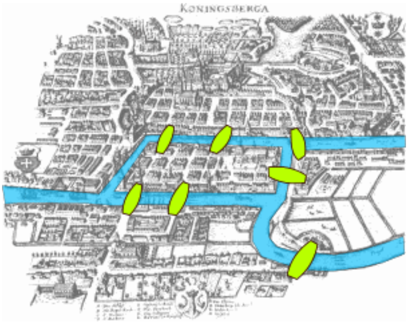

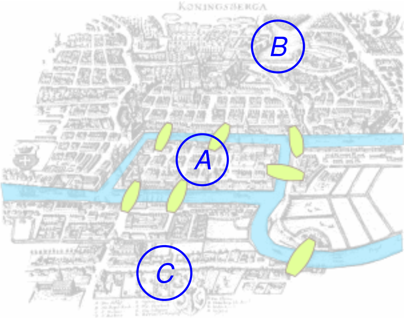

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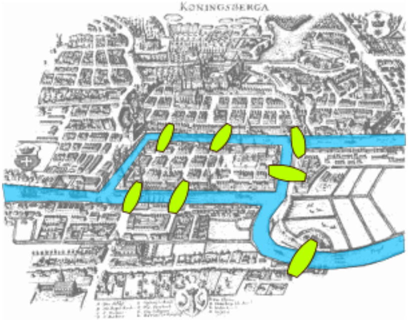

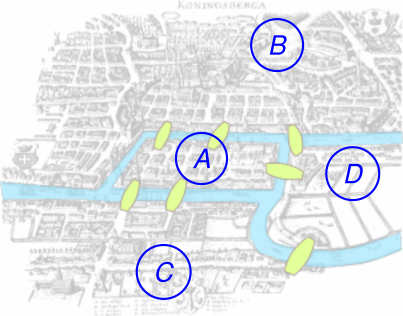

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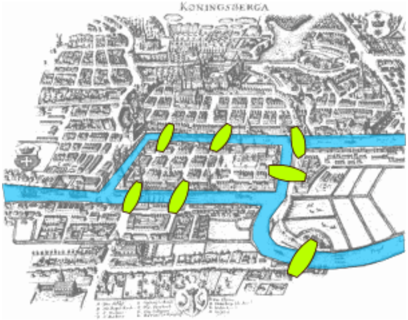

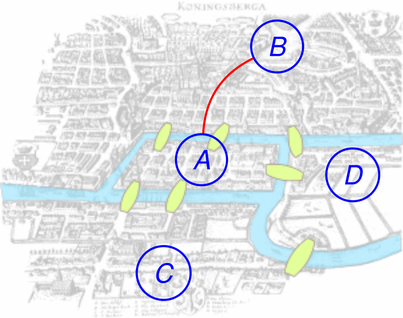

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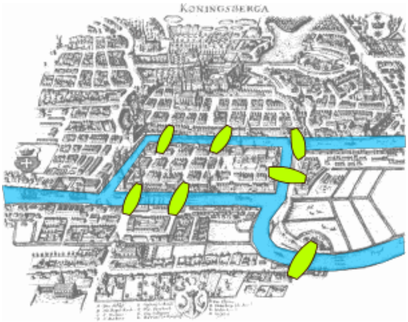

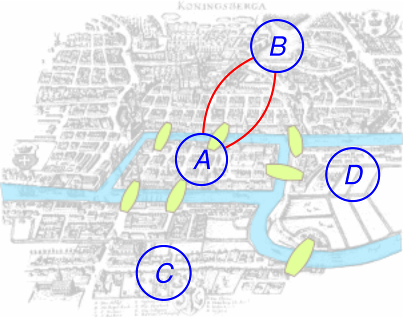

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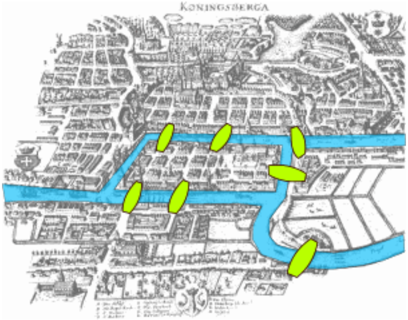

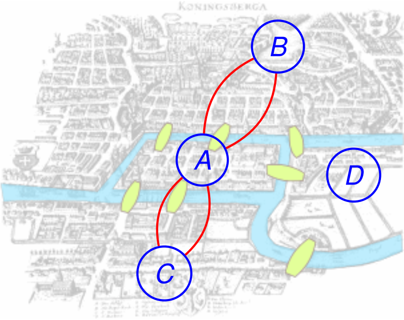

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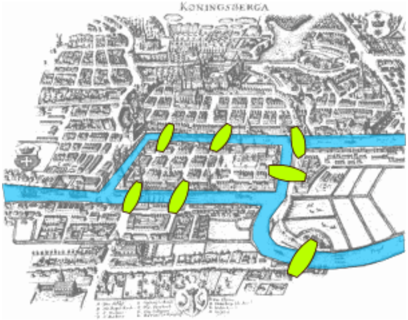

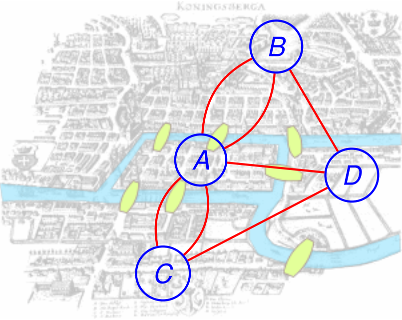

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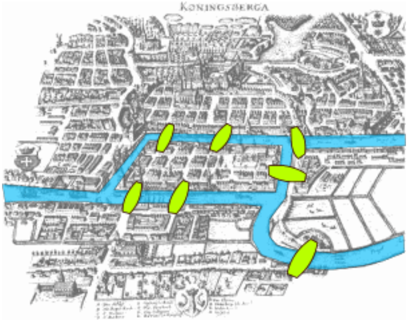

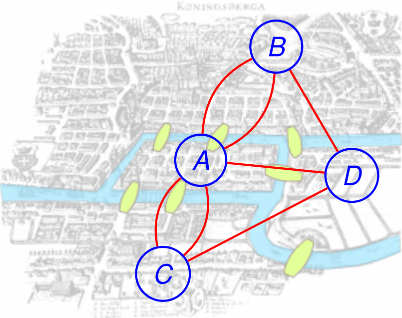

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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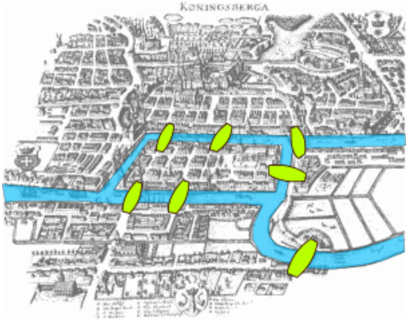

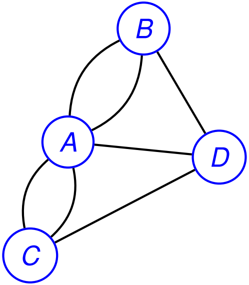

Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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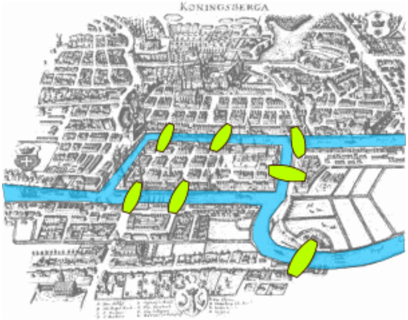

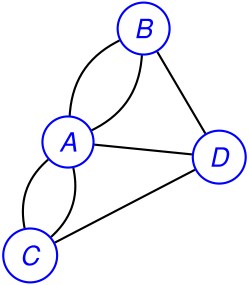

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Yes? No?

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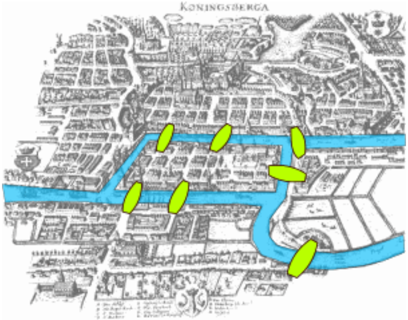

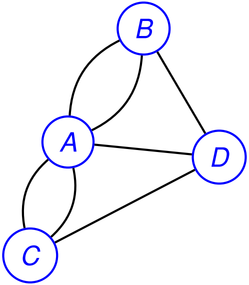

Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

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Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

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Proof of only if: Eulerian $\implies$ connected and all even degree.

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Eulerian Tour is connected so graph is connected.

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Tour enters and leaves vertex v on each visit.

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Uses two incident edges per visit.

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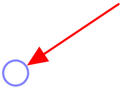

When you enter,

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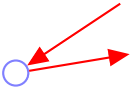

When you enter, you can leave.

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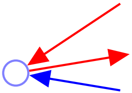

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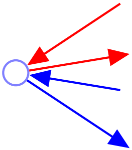

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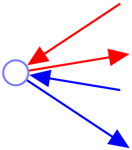

When you enter, you can leave.
For starting node,

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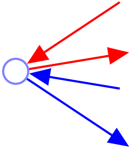

When you enter, you can leave.
For starting node, tour leaves first

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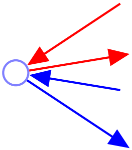

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.

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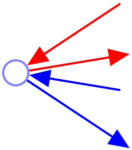

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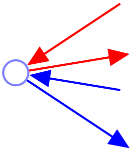

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Not The Hotel California.

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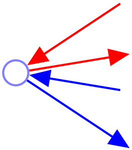

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(Timestamp: 4:02).

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

Finding a tour!

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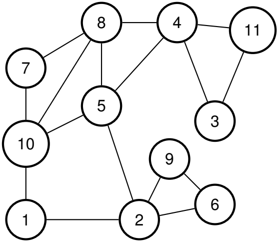

1. Take a walk starting from v (1) on "unused" edges

Finding a tour!

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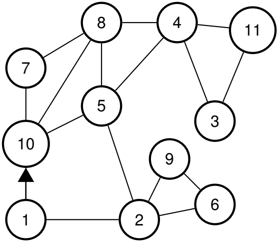

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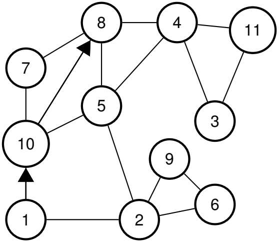

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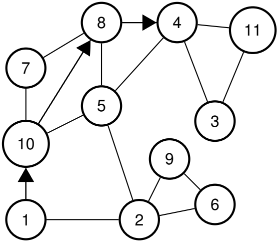

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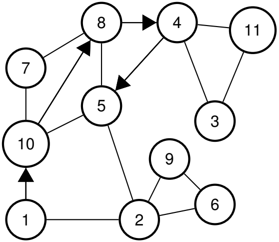

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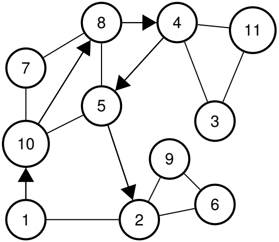

1. Take a walk starting from v (1) on “unused” edges

Finding a tour!

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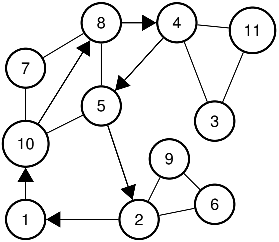

1. Take a walk starting from v (1) on "unused" edges
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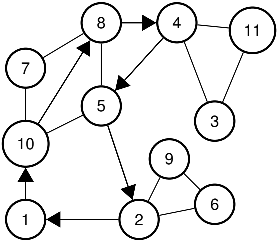

1. Take a walk starting from v (1) on "unused" edges
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2. Remove tour, C.

Finding a tour!

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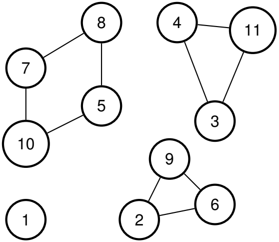

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2. Remove tour, C.
3. Let $G_1, \ldots, G_k$ be connected components.

Finding a tour!

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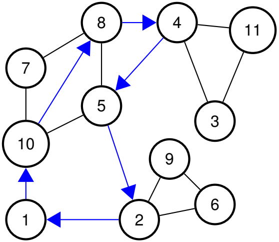

1. Take a walk starting from v (1) on "unused" edges
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3. Let $G_1, \ldots, G_k$ be connected components.
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Finding a tour!

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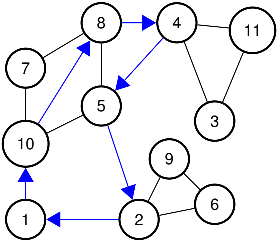

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Finding a tour!

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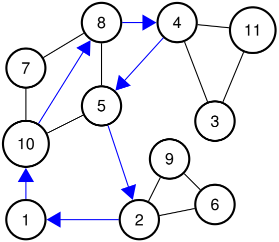

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   Each is touched by C.
      Why? G was connected.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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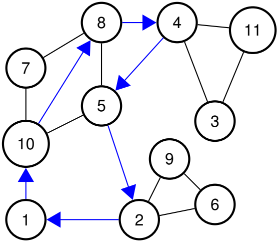

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Finding a tour!

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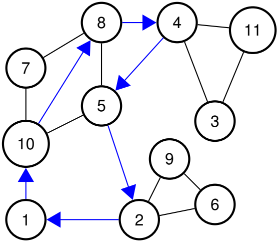

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3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$,

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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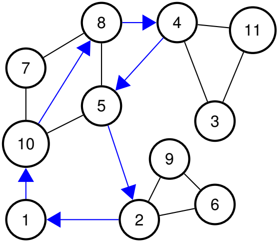

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   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$,

Finding a tour!

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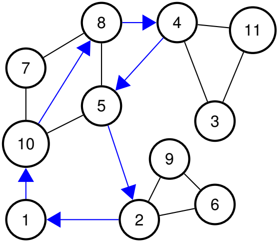

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3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

Finding a tour!

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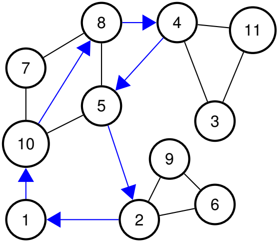

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      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.

Finding a tour!

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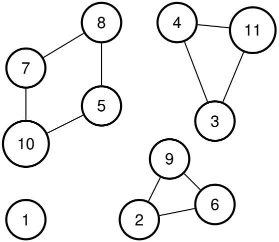

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4. Recurse on $G_1, \ldots, G_k$ starting from v_i

Finding a tour!

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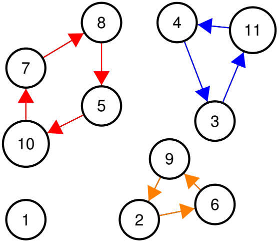

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Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
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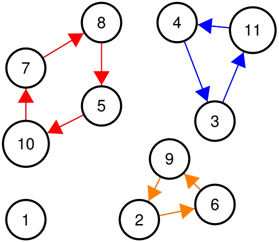

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   Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.

Finding a tour!

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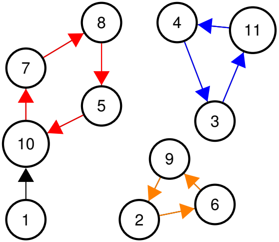

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5. Splice together.
   1,10

Finding a tour!

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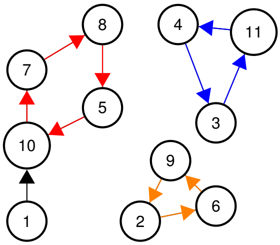

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   1,10,7,8,5,10

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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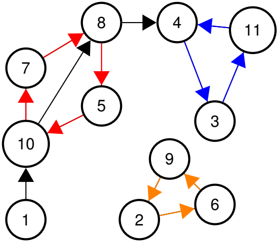

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   1,10,7,8,5,10 ,8,4

Finding a tour!

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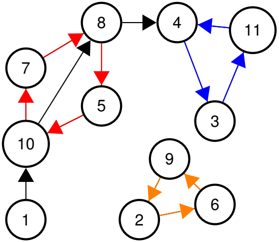

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5. Splice together.
   1,10,7,8,5,10 ,8,4,3,11,4

Finding a tour!

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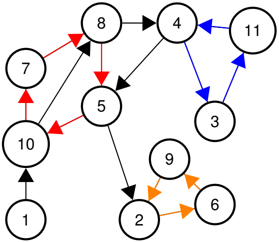

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5. Splice together.
   1,10,7,8,5,10 ,8,4,3,11,4 5,2

Finding a tour!

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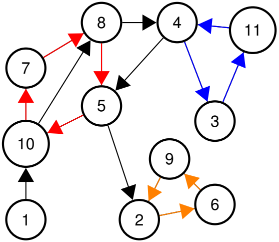

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2

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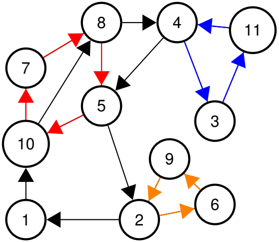

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Proof of Claim: Even degree. If enter, can leave except for v. □

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Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\Longrightarrow$
a vertex in G_i must be incident to a removed edge in C.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. □

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
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G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. □

3. Find tour T_i of G_i

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. □

3. Find tour T_i of G_i starting/ending at v_i.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. □

3. Find tour T_i of G_i starting/ending at v_i. Induction.

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(C) There is no hotel california in this graph.
(D) After removing a set of edges E' in a connected graph, every connected component is incident to an edge in E'
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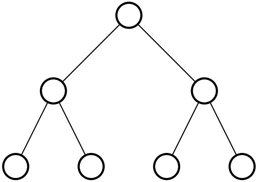
Only (F) is false.

A Tree, a tree.

Graph $G = (V, E)$.
Binary Tree!



More generally.

Trees.

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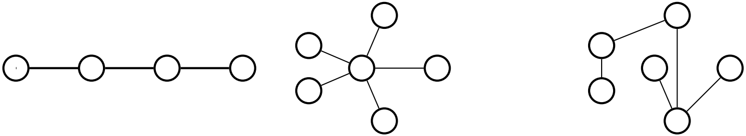

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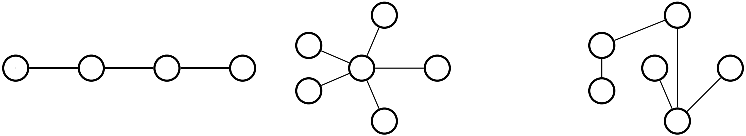

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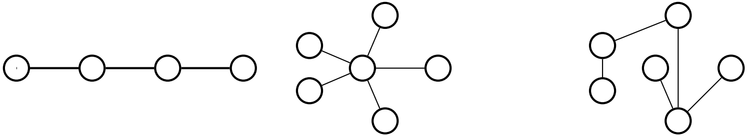

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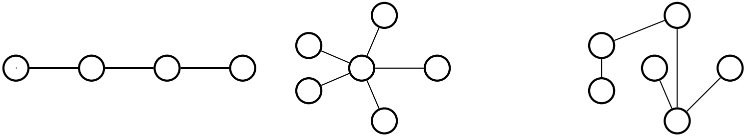

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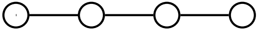

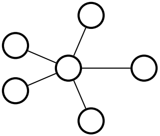

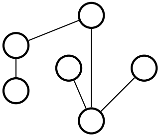

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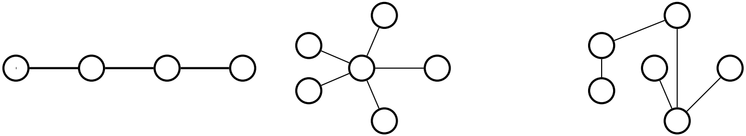

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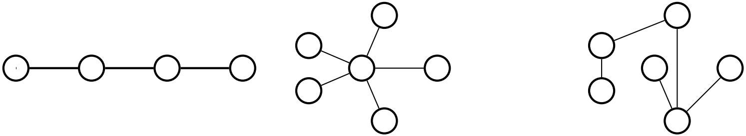

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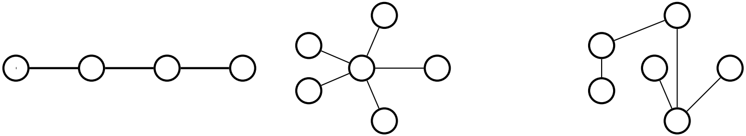

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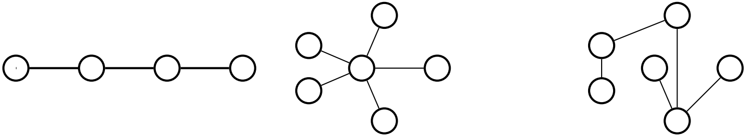

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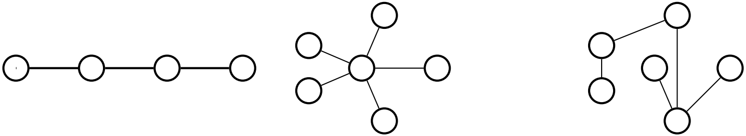

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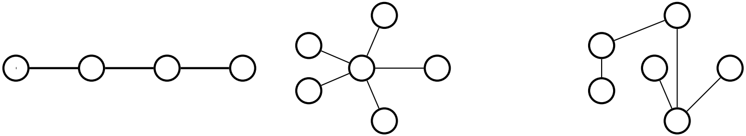

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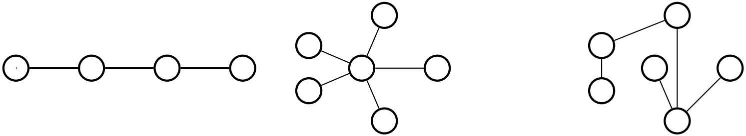

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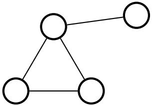

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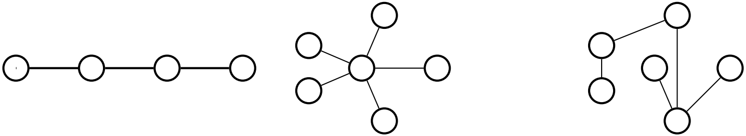

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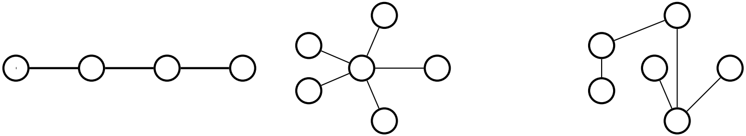

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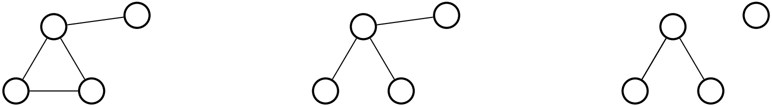

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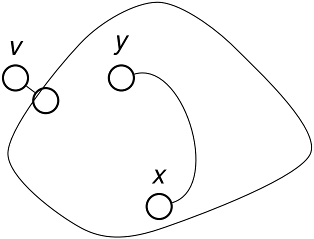

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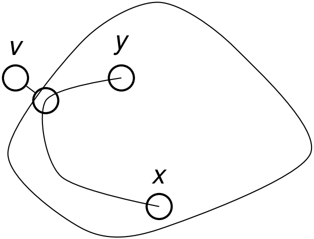

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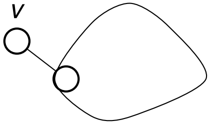

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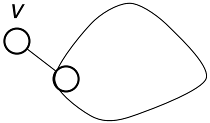

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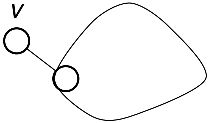

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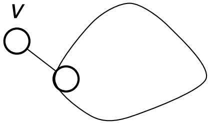

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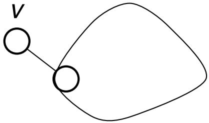

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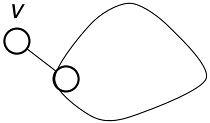

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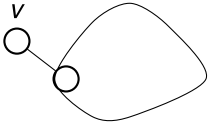
Proof of only if.



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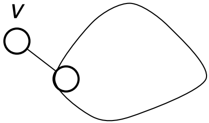

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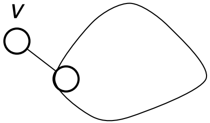

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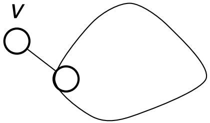
Proof of $\implies$: By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|E|=2(|V|-1)=2|V|-2$
Average degree $(2|V|-2)/|V|=2-(2/|V|)$.

Proof of only if.



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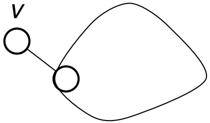

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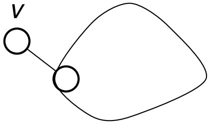

Thm:
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Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

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Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
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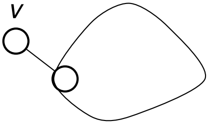

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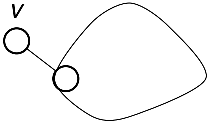

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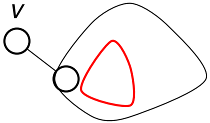

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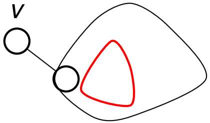

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And no cycle in G since degree 1 cannot participate in cycle.

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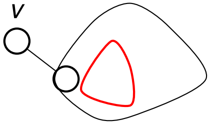

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Proof of if

Thm:
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Proof of if

Thm:
"G is connected and has no cycles"
$\Longrightarrow$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.

Proof of if

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Until get stuck.

Proof of if

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Can't visit more than once since no cycle.

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Entered.

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Entered. Didn't leave.

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Removing node doesn’t create cycle.

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New graph is connected.

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By induction $G-v$ has $|V|-2$ edges.
G has one more or $|V|-1$ edges.

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Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ edges.

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V|-1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2-2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
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(B), (C), (D) are true

Lecture Summary.

Graphs.

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  Basics.

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Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.

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maximal set of vertices that are connected.

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- Take a walk until stuck to form tour.

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$n-1$ vertices, $n-2$ edges and connected $\implies$ acyclic.
(Ind. Hyp.)
degree 1 vertex is not in a cycle.
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